

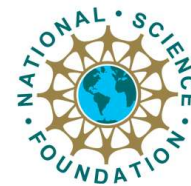
# History of the USD Shell Model Interactions

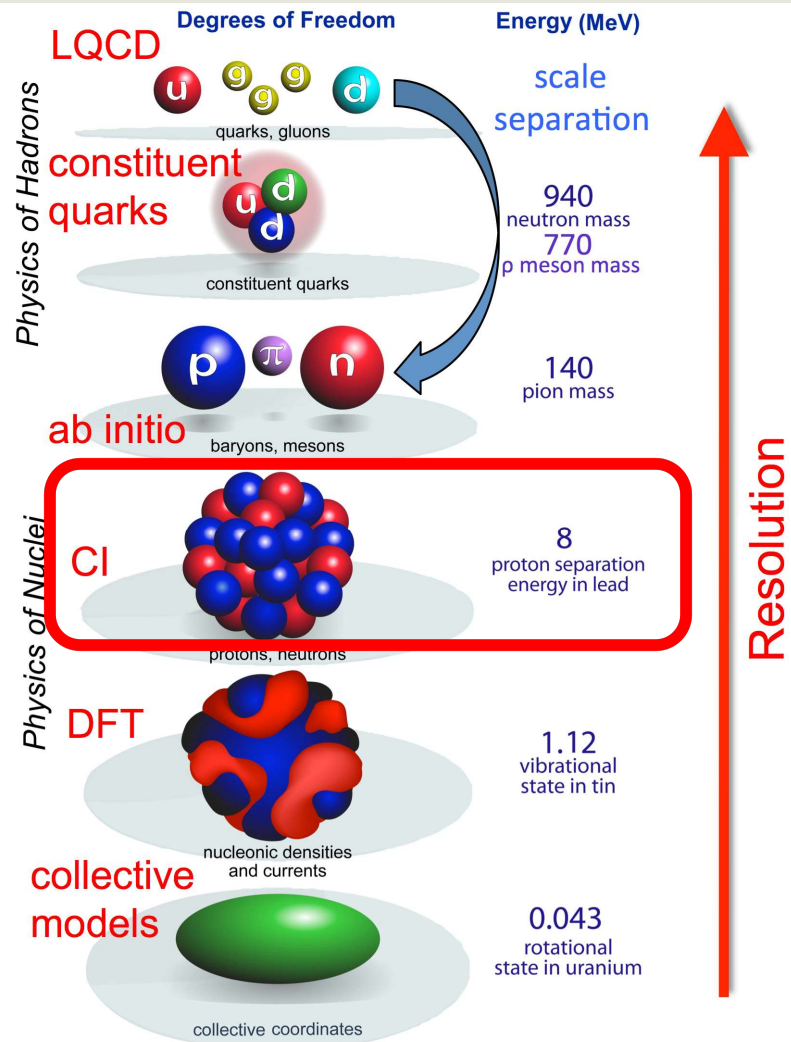
Alex Brown

Xth Tastes of Nuclear Physics

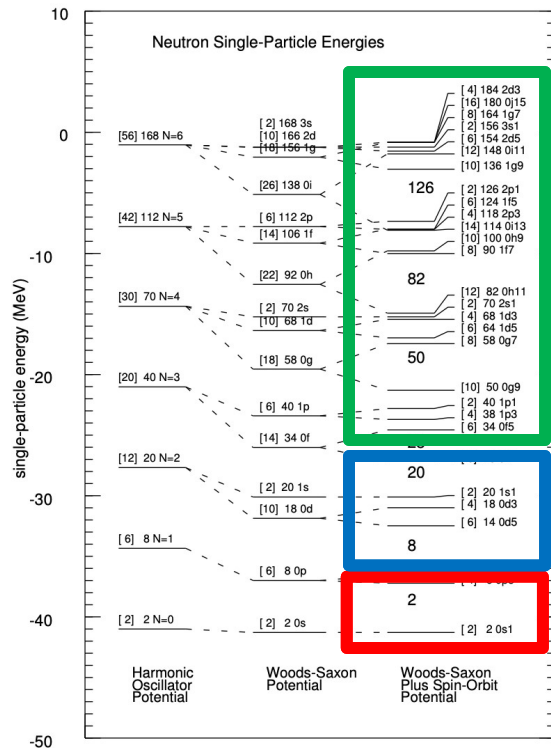
University of the Western Cape. South Africa

December 4<sup>th</sup>, 2020

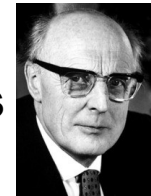




# The Nuclear Shell Model



Developed independently by  
**Maria Goeppert Mayer and Hans Jensen**  
*(Nobel Prize in Physics 1963)*



Nucleons sitting in a mean field generated by all nucleons naturally form the nuclear “shells”

Group orbits into three categories:

- High lying orbitals: empty
- sd model Space: where the many body problem is solved involves the  $1s_{1/2}$ ,  $0d_{5/2}$  and  $0d_{3/2}$  orbitals
- Inert Core: filled

$$H|\Psi\rangle = E|\Psi\rangle \rightarrow H_{SM}|\Psi'\rangle = E|\Psi'\rangle$$

$$|\Psi'\rangle = \sum c_{\alpha} |\phi_{\alpha}\rangle$$

# History

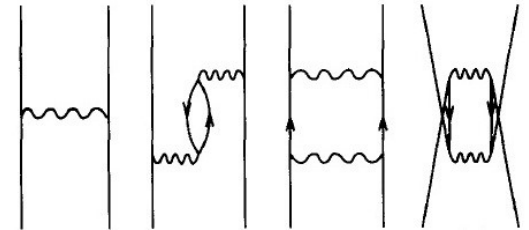
Break through idea – Tom Kuo and Gerry Brown (1968...)

introduction two-body matrix elements (TBME) derived from nucleon-nucleon scattering potentials

G matrix to handle the short-range correlations

Core-polarization to take into account the truncation to the sd shell.

Applied to two-particles ( $A=18$ ) and two holes ( $A=38$ ).



Break through idea – Wilton Chung and Hobson Wildenthal (1976-1982)

Used many-body techniques to apply these two-body matrix elements to a wide range of nuclei

Found that the spectra could be improved with small changes to the TBME obtained by constraints to some experimental energy data.

EMPIRICAL RENORMALIZATION OF SHELL-MODEL  
HAMILTONIANS and MAGNETIC DIPOLE  
MOMENTS OF sd-SHELL NUCLEI

By

Wilton Chung

A DISSERTATION

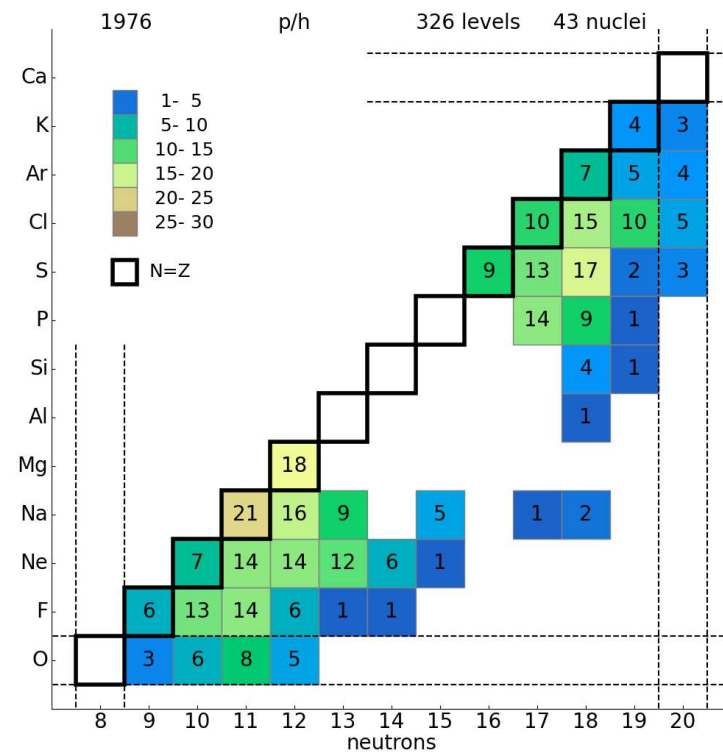
Submitted to  
Michigan State University  
in partial fulfillment of the requirements  
for the degree of

DOCTOR OF PHILOSOPHY

Department of Physics

1976

(1976) Two different Hamiltonians  
“particle” for the lower end (CWP)  
“hole” for the higher end (CWH)



1976-1982 – computers got better and a lot more work by Wildenthal resulted a “universal” Hamiltonian for the entire sd shell - **USD**

Success related to the introduction of a smooth mass dependence (but turns out this was not the main the main reason)

Reported as an abstract in the Bulletin of the American Physical Society 27, 725 (1982)

No Coulomb - isospin was conserved

Rms deviation in the difference between experimental and theoretical excitation energies was about 180 keV

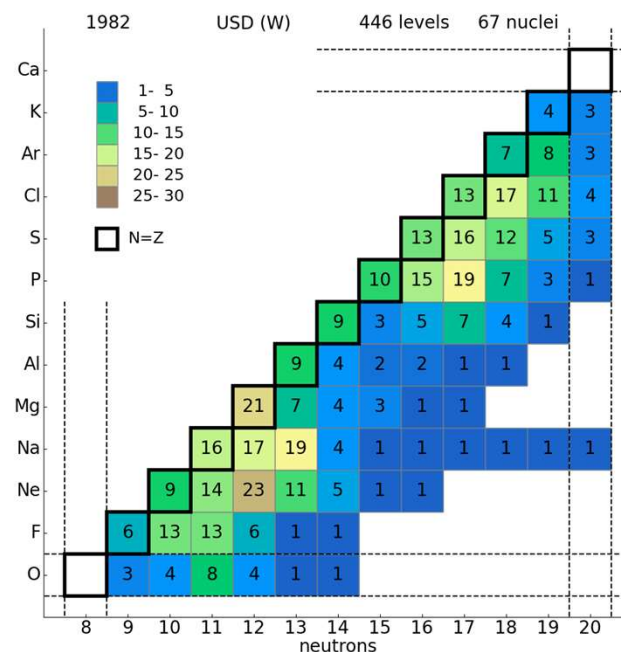
Usual reference for USD in this conference proceeding:

Empirical Strengths of Spin Operators in Nuclei

B. H. Wildenthal

Proceedings of the International School of Nuclear Physics, Erice, Sicily

*Progress in Particle and Nuclear Physics*, Vol.11, Pergammon Press, Oxford, (1984) 5-51; **4-6-1983**



1982 – 1988

Erich Ormand - added Coulomb and isospin interactions as a perturbation on USD – call **USD<sub>CD</sub>** – (**charge dependent**) applied in a proton-neutron (pn) basis

Alex Brown and others made many applications

All of this was summarized in a review article often used as a reference of USD (1000 citations)

Annual Review of Nuclear and Particle Science 38, 29 (1988)

## STATUS OF THE NUCLEAR SHELL MODEL

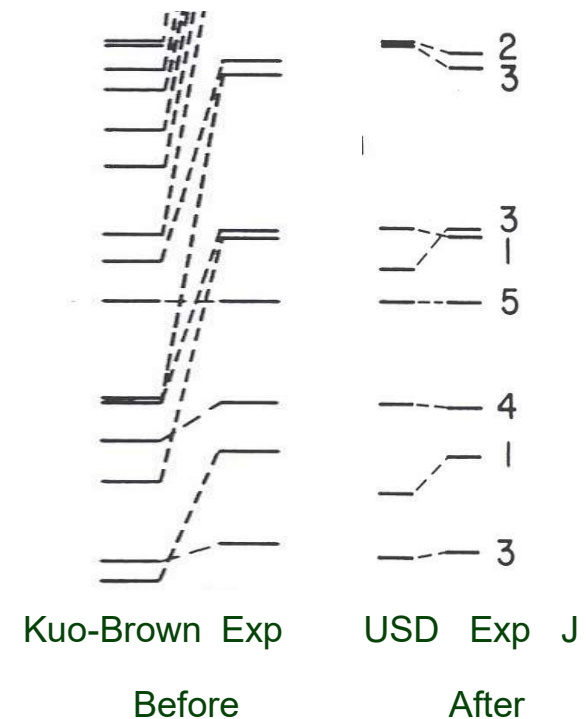
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$^{22}\text{Na}$



ANNALS OF PHYSICS **182**, 191–236 (1988)

## Semi-empirical Effective Interactions for the $1s-0d$ Shell

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AND

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Received December 7, 1987

1986-1988

Can we find a “universal” interaction that goes beyond the  $sd$  shell?

Spin-tensor decomposition of 63 TBME into central (20), tensor (16), spin-orbit (9) and anti-symmetric spin-orbit (18).

One-boson exchange potentials

“best-fit” interaction based on a 14-parameter density-dependent two-body potential with an rms of 260 keV.

We could not find a “universal” interaction of this type that worked for  $sd$  and  $pf$  at the same time.

Comment: what we call core ( $^{16}\text{O}$ ) and single-particle energies have a combination of kinetic and potential contributions. In these type of fits we do not know the decomposition. The wavefunctions only depend on the total. But the interpretation of the TBME in terms of a potential depend on this.



1976-1982 USD had been arrived at by many iterations of the single-valued decomposition (SVD) method.

The details were never fully published

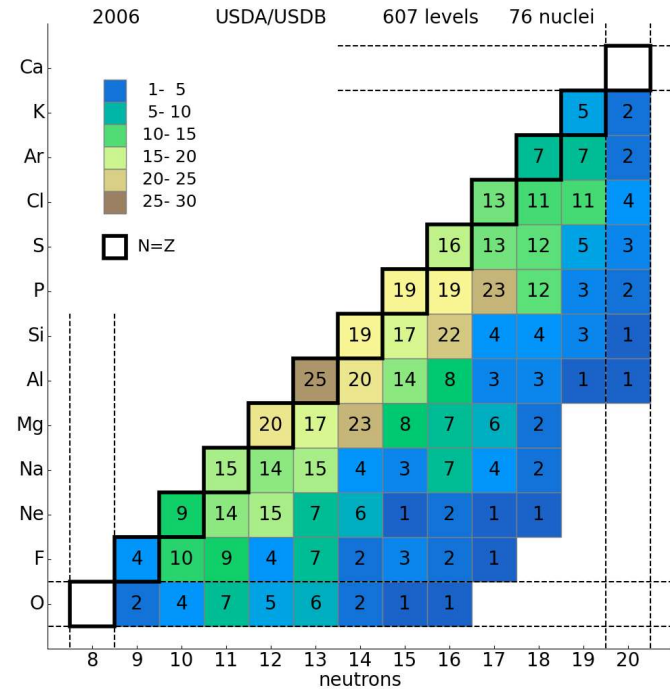
Around 2004 I decided that this should be “done properly”

Starting NN based TBME from Morten Hjorth-Jensen

Werner Richter helped to put together an complete set of data.

Two new isopin-conserving Hamiltonians were obtained  
**USDA** (30 linear combination of SVD parameters) and  
**USDB** (56 linear combinations of SVD parameters)

Ormand's CD was added to make **USDACD** and  
**USDBCD**.



PHYSICAL REVIEW C **74**, 034315 (2006)

### New “USD” Hamiltonians for the *sd* shell

B. Alex Brown<sup>1</sup> and W. A. Richter<sup>2</sup>

<sup>1</sup>*Department of Physics and Astronomy, and National Superconducting Cyclotron Laboratory, Michigan State University, East Lansing, Michigan 48824-1321, USA*

<sup>2</sup>*Department of Physics, University of the Western Cape, Private Bag X17, Bellville 7530, South Africa*

(Received 23 May 2006; published 13 September 2006)

2018... Aaron Magilligan completely rewrote the codes for the single-valued decomposition (SVD) method in a proton-neutron basis using outputs from NuShellX.

Coulomb and isospin non-conserving interactions were included explicitly - resulted in **USDC** and **USDCm**

He also used the in-medium renormalization group (IMSRG) results for  $^{28}\text{Si}$  from Ragnar Stroberg et al. as a starting point **USDI** and **USDI<sub>m</sub>**

PHYSICAL REVIEW C **101**, 064312 (2020)

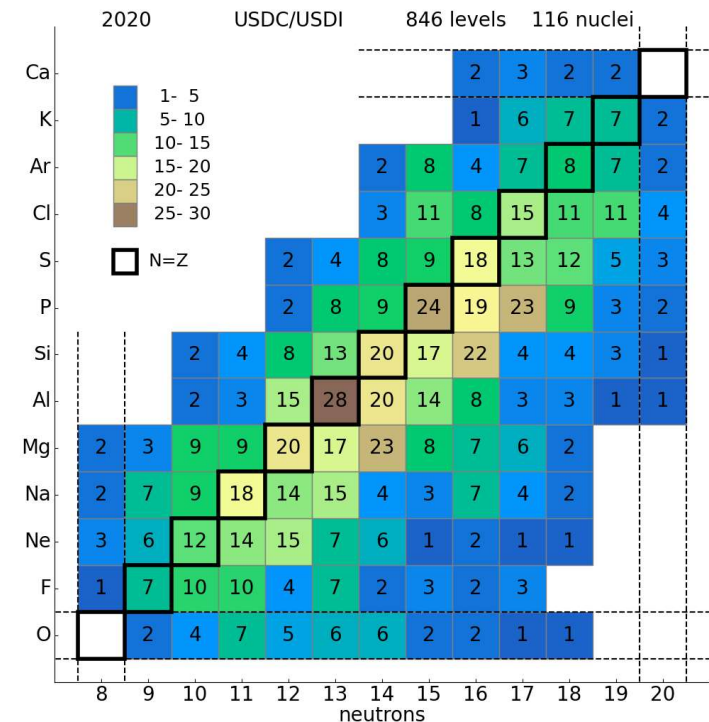
### New isospin-breaking “USD” Hamiltonians for the *sd* shell

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*Department of Physics and Astronomy and National Superconducting Cyclotron Laboratory,  
and Michigan State University, East Lansing, Michigan 48824, USA*



(Received 17 December 2019; accepted 22 May 2020; published 15 June 2020)

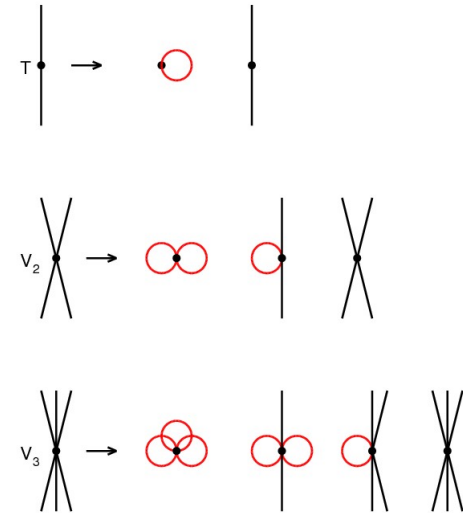


## 2017..... Development of nucleus-dependent Hamiltonians

Example above with the In medium similarity renormalization group (IMSRG) method

Include three-body interactions averaged over core - this improves in the energy spectra.

$$H = E_0 + \sum_{ij} f_{ij} \{a_i^\dagger a_j\} + \frac{1}{4} \sum_{ijkl} \Gamma_{ijkl} \{a_i^\dagger a_j^\dagger a_l a_k\} + \frac{1}{36} \sum_{ijklmn} W_{ijklmn} \{a_i^\dagger a_j^\dagger a_k^\dagger a_n a_m a_l\},$$



PRL **118**, 032502 (2017)

PHYSICAL REVIEW LETTERS

week ending  
20 JANUARY 2017

### Nucleus-Dependent Valence-Space Approach to Nuclear Structure

S. R. Stroberg,<sup>1,\*</sup> A. Calci,<sup>1</sup> H. Hergert,<sup>2</sup> J. D. Holt,<sup>1</sup> S. K. Bogner,<sup>2</sup> R. Roth,<sup>3</sup> and A. Schwenk<sup>3,4,5</sup>

ARNPS 69, 307 (2019)

At present the rms deviation is on the order of 650 keV

About 4 times larger than USD

These type of calculations will be improved.

*Annual Review of Nuclear and Particle Science*

## Nonempirical Interactions for the Nuclear Shell Model: An Update

S. Ragnar Stroberg,<sup>1,2,3</sup> Heiko Hergert,<sup>4</sup>  
Scott K. Bogner,<sup>4</sup> and Jason D. Holt<sup>1</sup>

