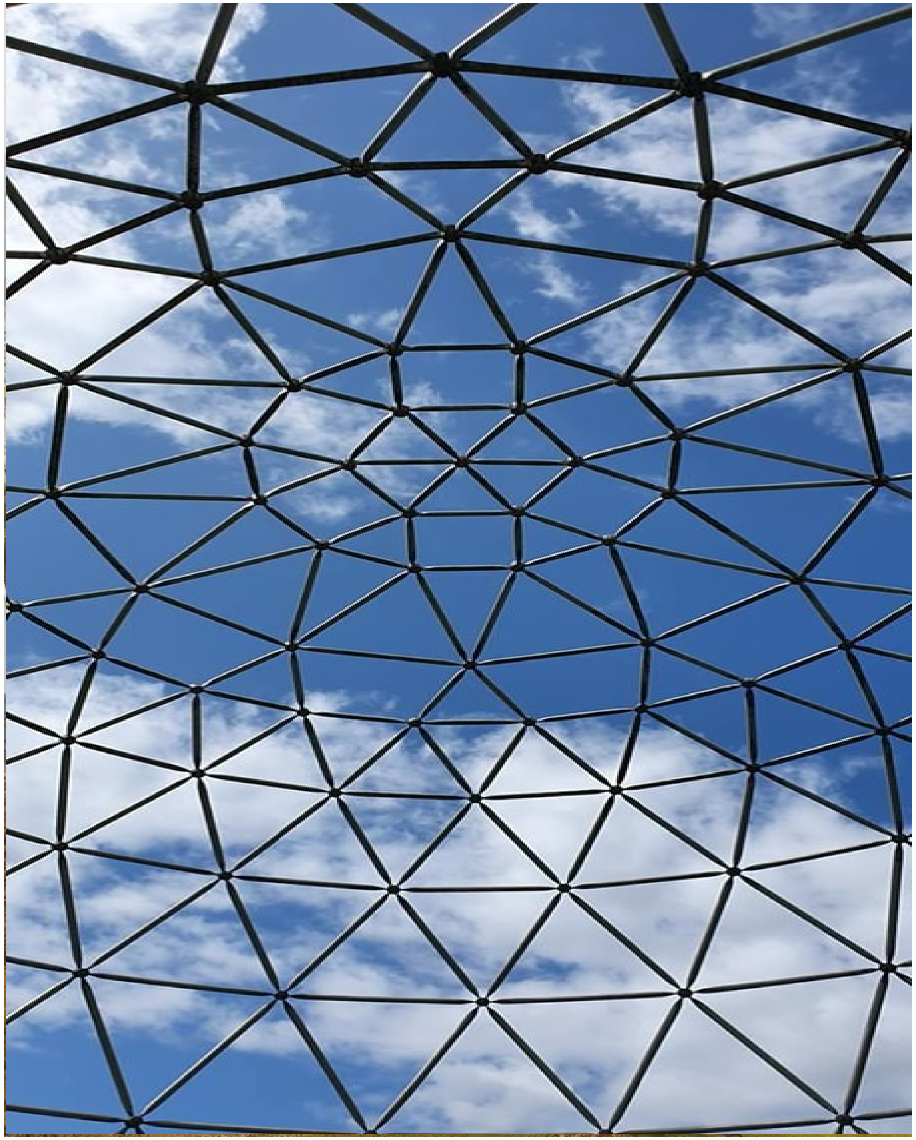
Describing low-energy nuclear reactions with quantum wave-packet dynamics



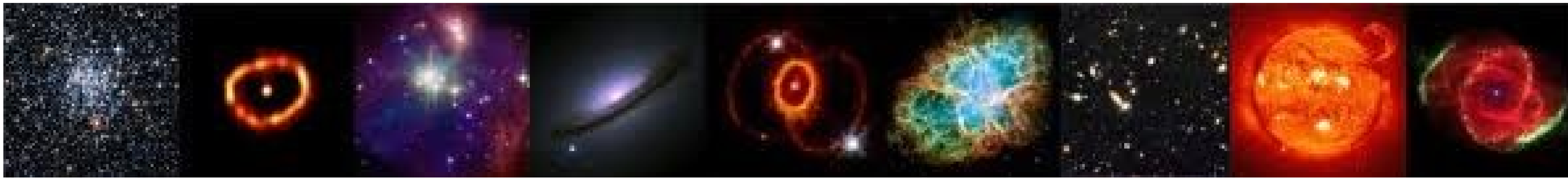
ALEXIS DIAZ-TORRES

UNIVERSITY OF SURREY

Examples

- $^{6}Li + ^{209}Bi$ (with Maddalena **Boselli**)
- $^{12}C + ^{12}C$ (with Michael **Wiescher**)
- $^{16}O + ^{154}Sm$ (with Terry **Vockerodt**)

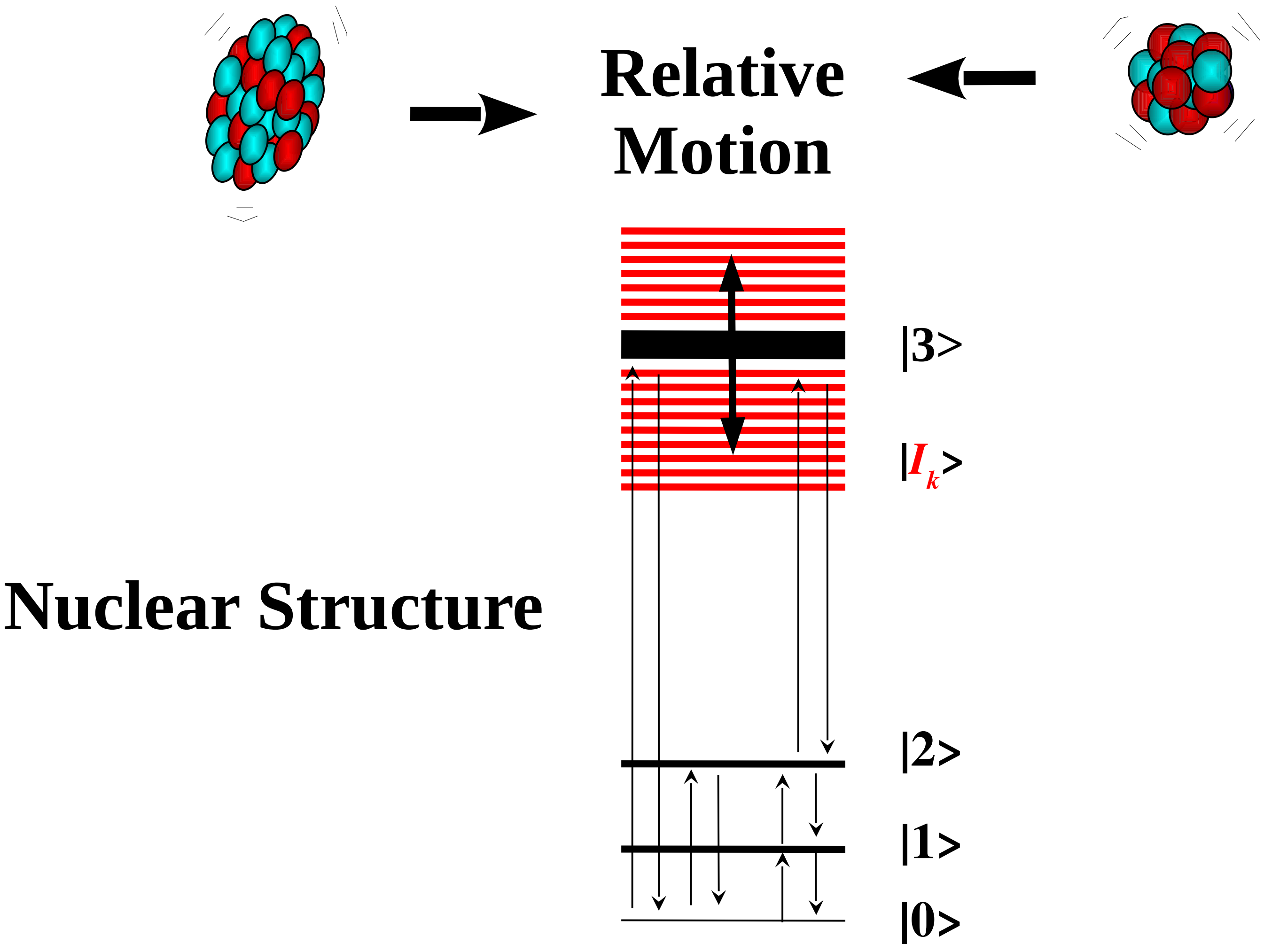
Importance of the Physics of Nuclear Reactions



- **This physics** is crucial for understanding **energy production** and **element creation** in the Universe.

- **Nuclear reactions** are the **primary probe** of the **New Physics**.

The Physics of Low-Energy Nuclear Reactions



Interplay between **nuclear structure** and **reaction dynamics** determines reaction outcomes (**cross sections**)

Quantum Wave-Packet Dynamics

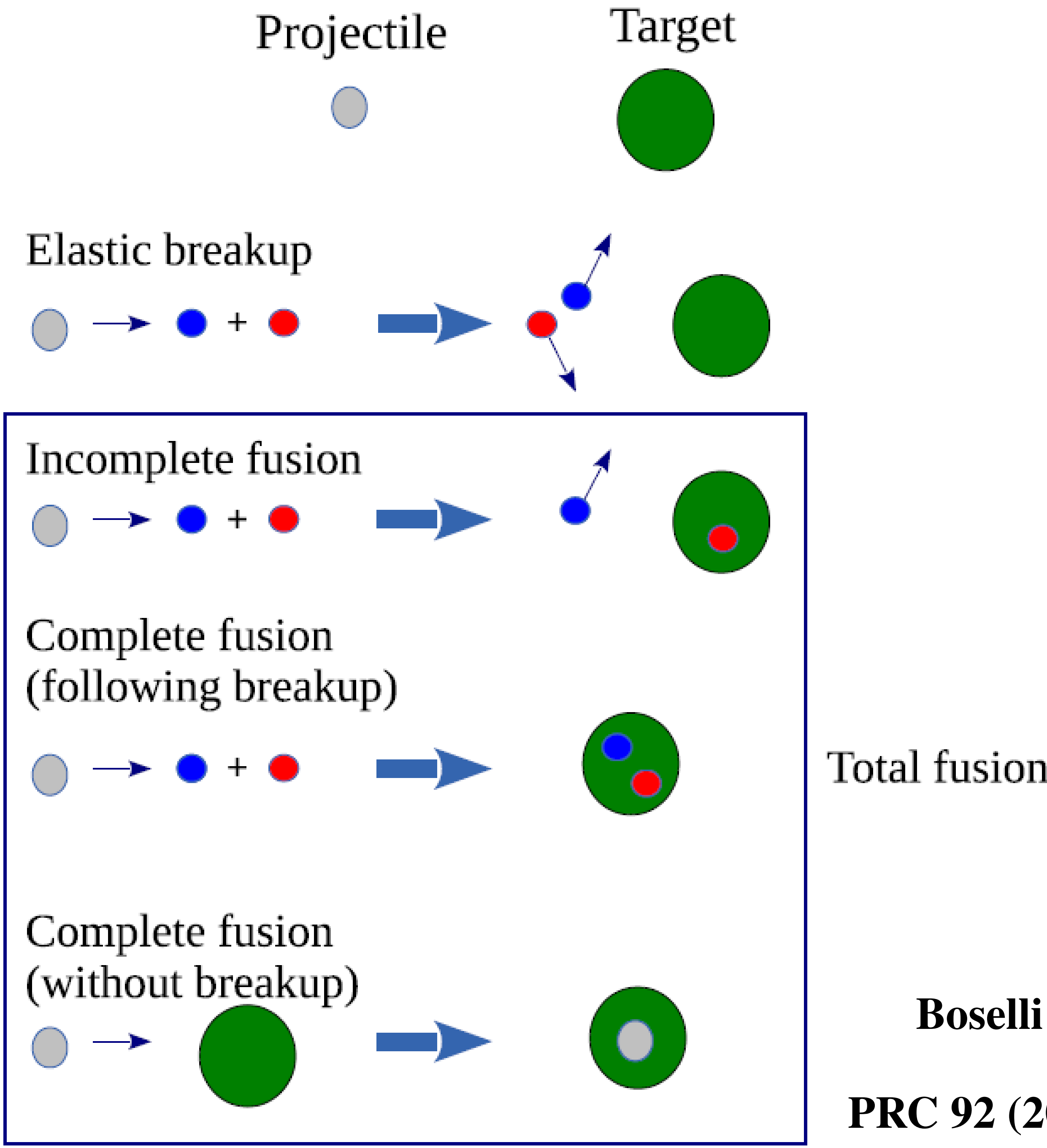
D.J. Tannor, Quantum Mechanics: a Time-Dependent Perspective, USB, 2007

- **Preparation:** the initial state $\Psi(t = 0)$

- **Time propagation:** $\Psi(0) \longrightarrow \Psi(t)$,
  guided by the operator, $\exp(-i\,\hat{H}\,t/\hbar)$
  $\hat{H}$ is the model Hamiltonian

- **Analysis:** extraction of probabilities from the time-dependent wave function

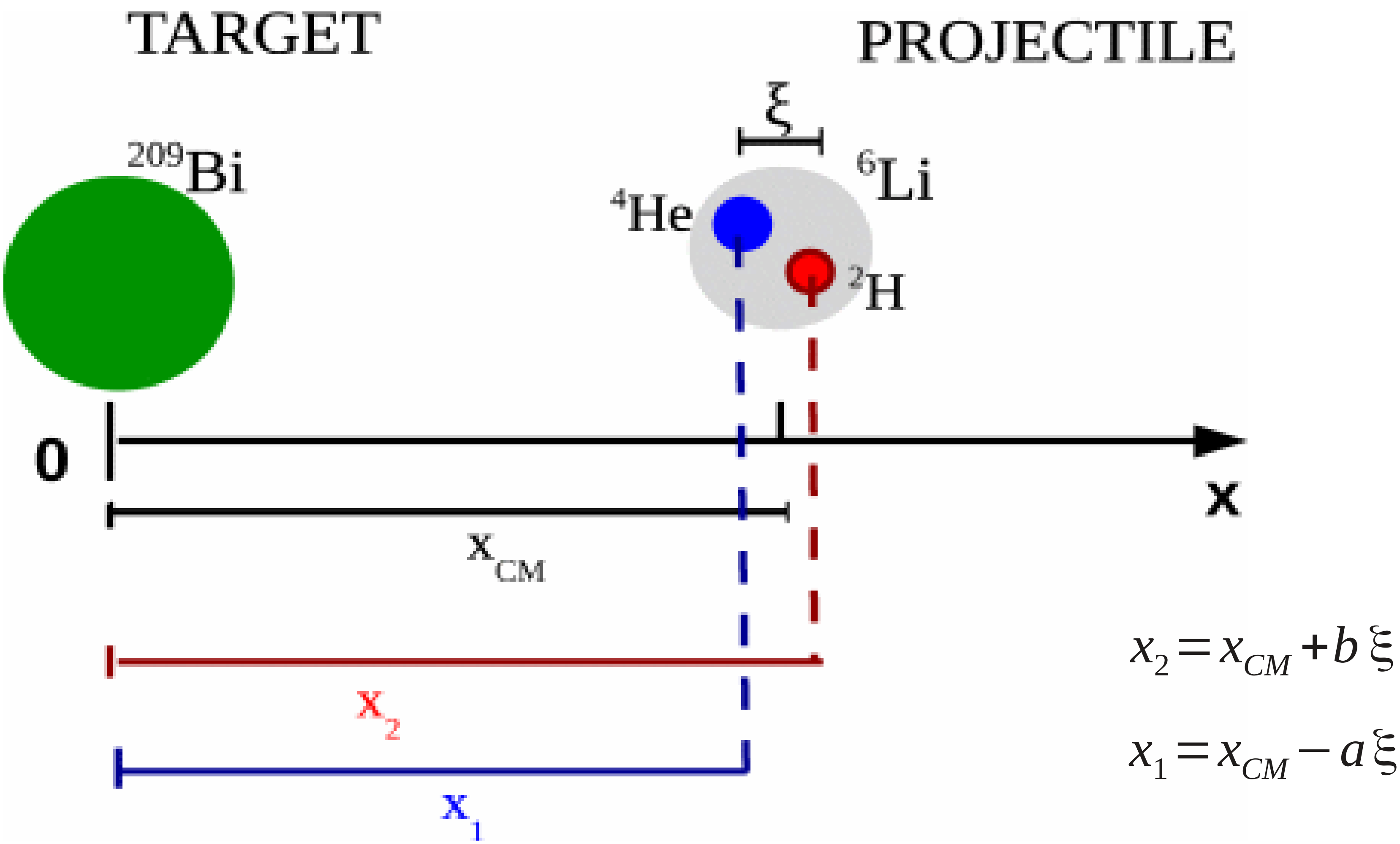
Lecture 1: Fusion Dynamics of Weakly Bound Nuclei



Boselli & AD-T,

PRC 92 (2015) 0446110

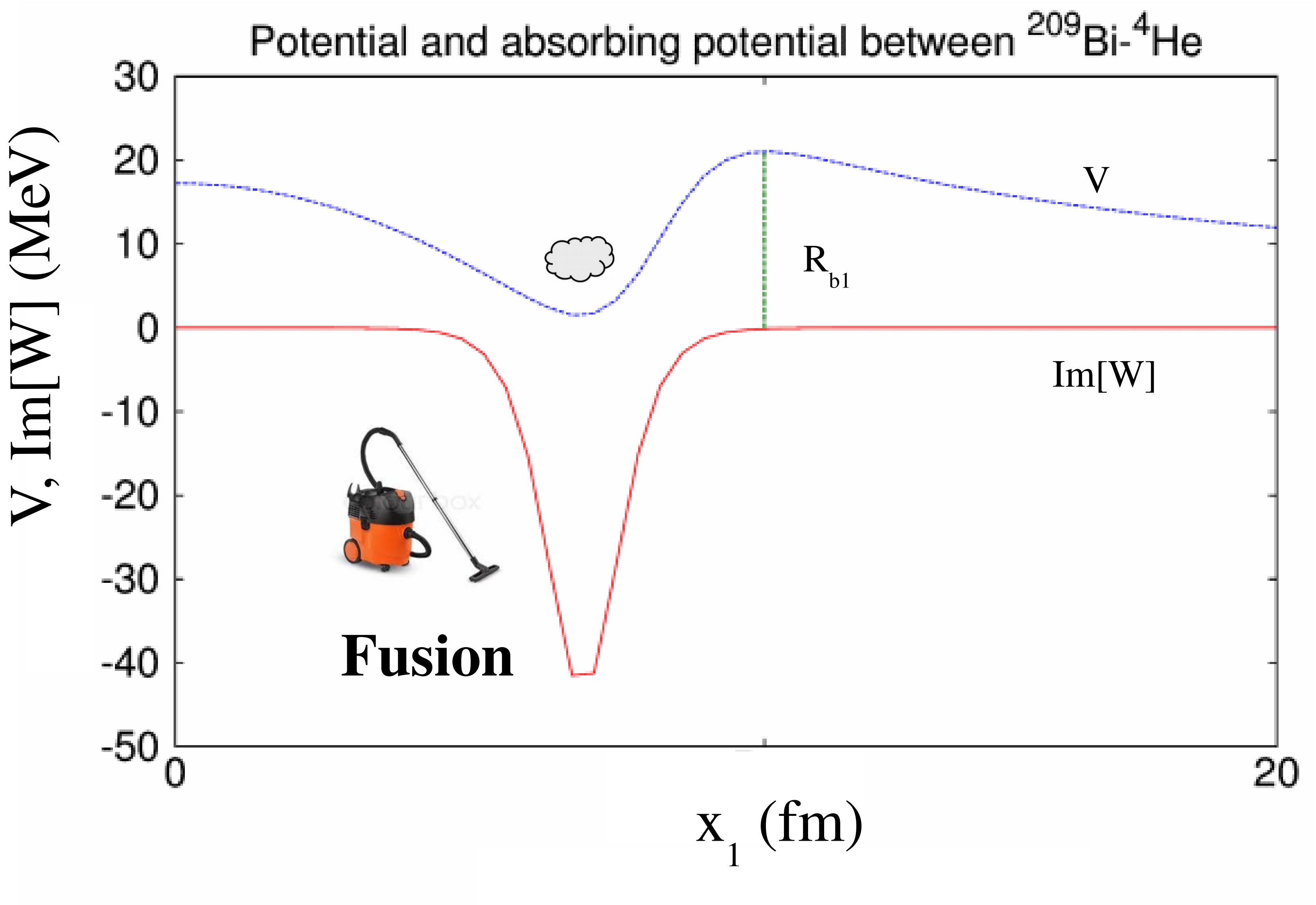
One-Dimensional Toy Model



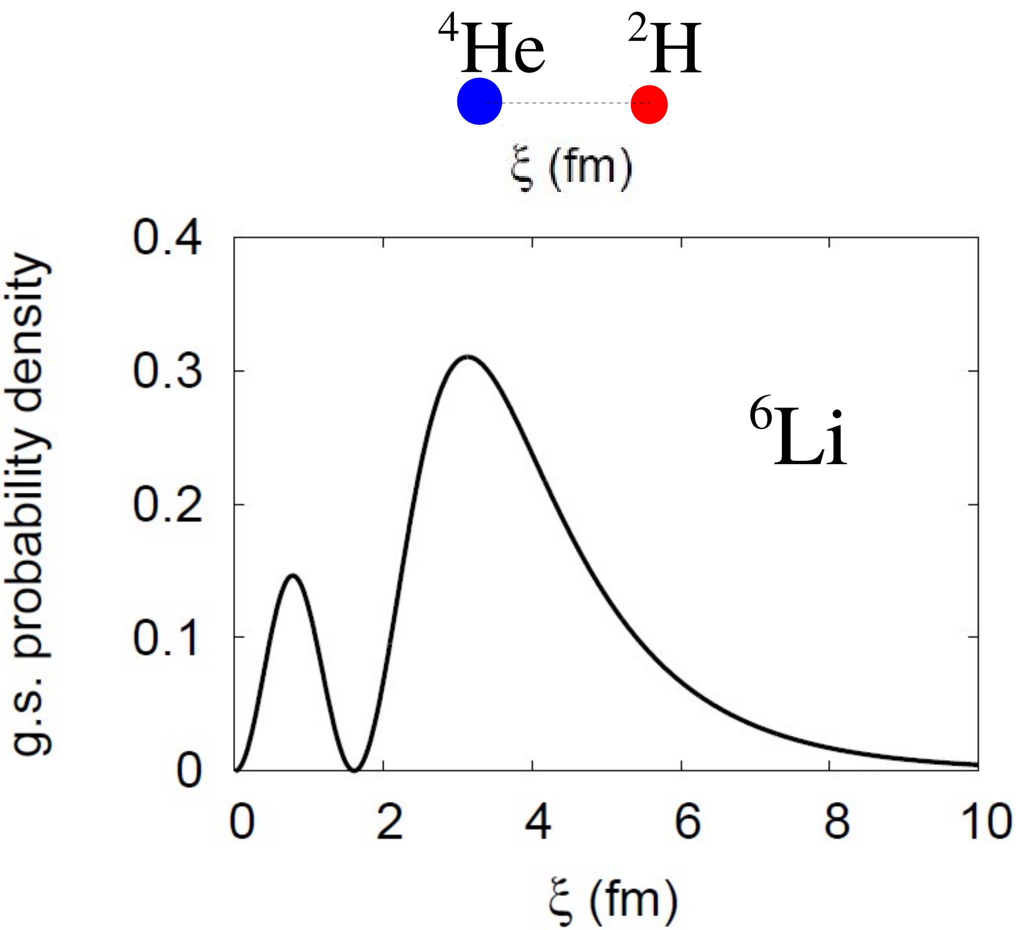
$$H = \frac{P_{x_{CM}}^2}{2M_{T12}} + \frac{P_{\xi}^2}{2m_{12}} + U_{12}(\xi) + V_{T1}(x_{CM} - a\xi) + V_{T2}(x_{CM} + b\xi)$$

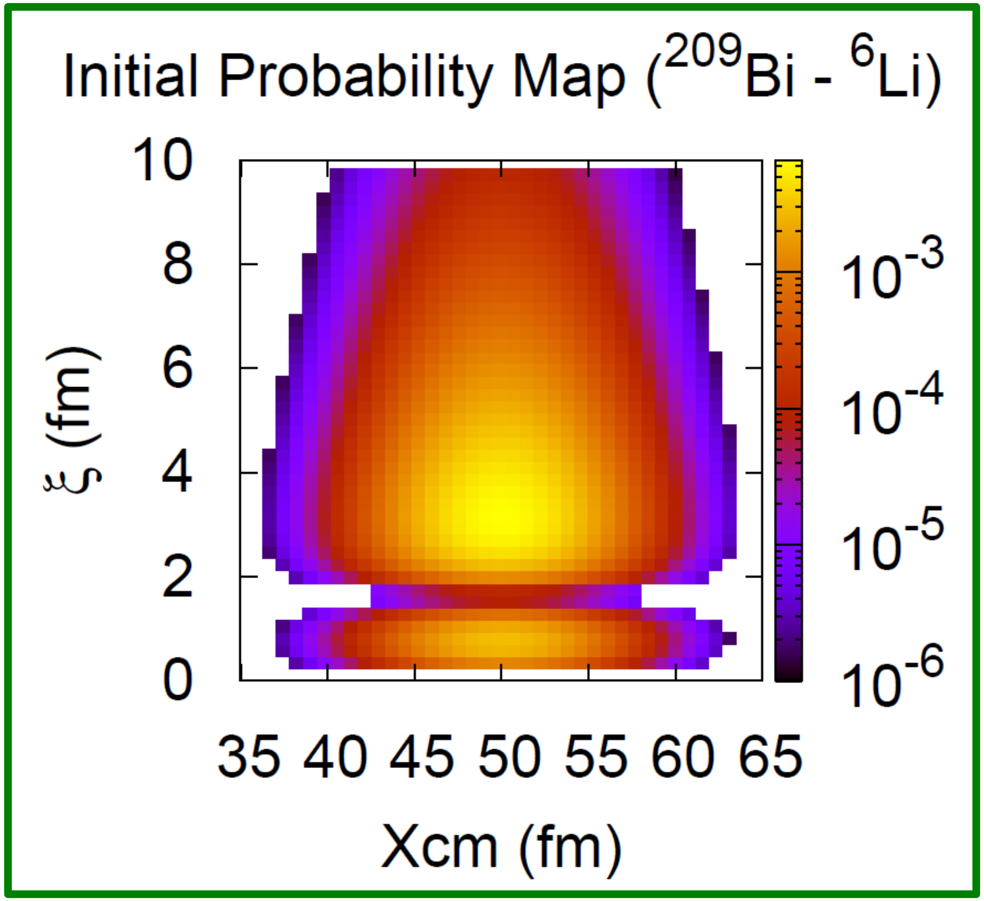
Describing Fusion

♦ To simulate fusion (**irreversibility**): acting inside the Coulomb barrier $-iW_{T1}(x_1)$ & $-iW_{T2}(x_2)$



Preparing the Initial State





Time Propagation

R. Kosloff, Ann. Rev. Phys. Chem. **45** (1994) 145

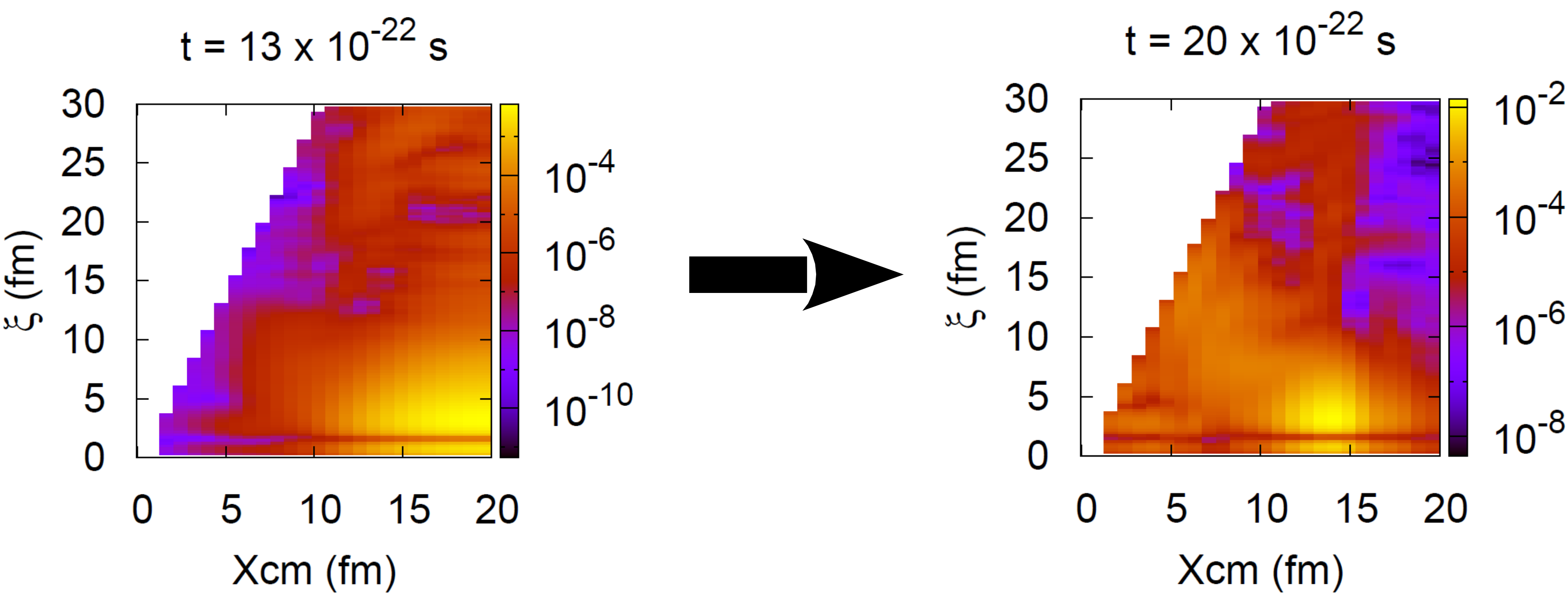
$$\Psi(t+\Delta t) = \exp\left(-i\frac{\hat{H}\,\Delta t}{\hbar}\right)\Psi(t)$$

$$\exp\left(-i\frac{\hat{H}\,\Delta t}{\hbar}\right) \approx \sum_n a_n\, Q_n(\hat{H}_{norm})$$

$$\hat{H}_{norm} = \frac{(\bar{H}\,\hat{1} - \hat{H})}{\Delta H}$$

The Chebyshev Propagator

$$a_n = i^n (2-\delta_{n0}) \exp(-i\frac{\bar{H}\,\Delta t}{\hbar})\, J_n(\frac{\Delta H\,\Delta t}{\hbar})$$

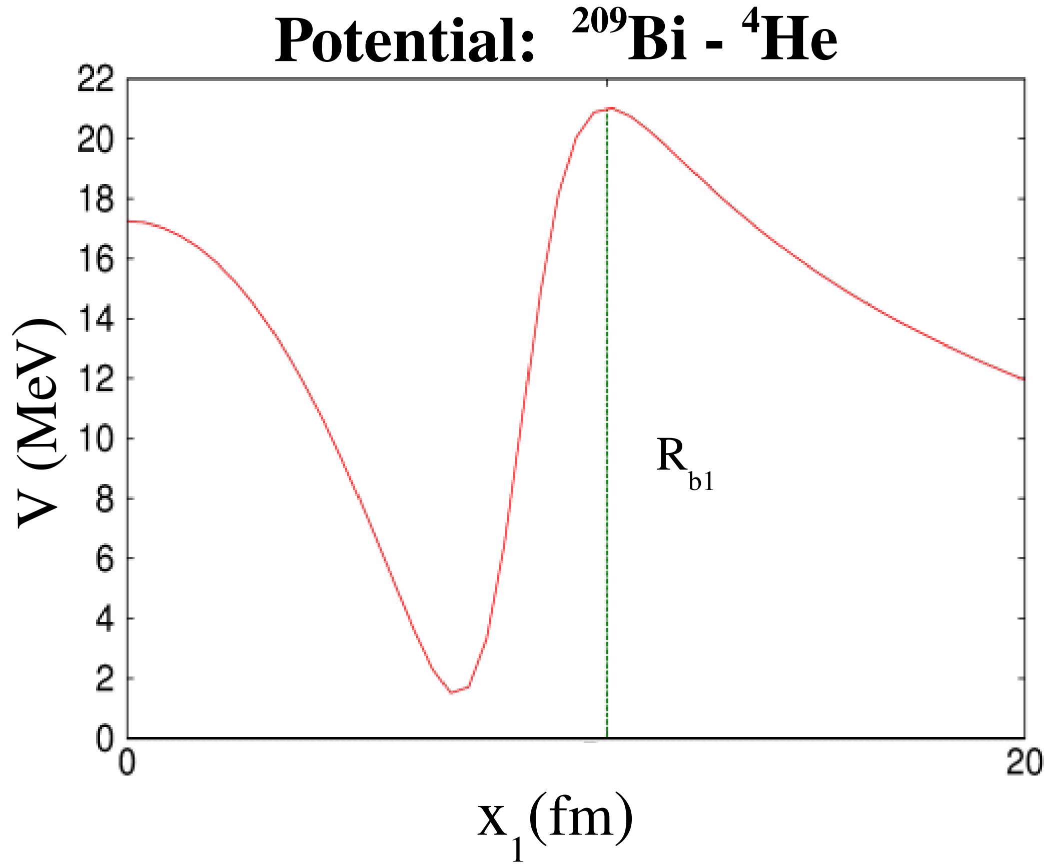


Analysis: Slicing the Wave Function

- Projection operator acting on the position x_i of the fragment relative to the target

(Heaviside function)

$$P_i = \Theta\left(R_{bi} - x_i\right)$$
$$Q_i = 1 - P_i$$



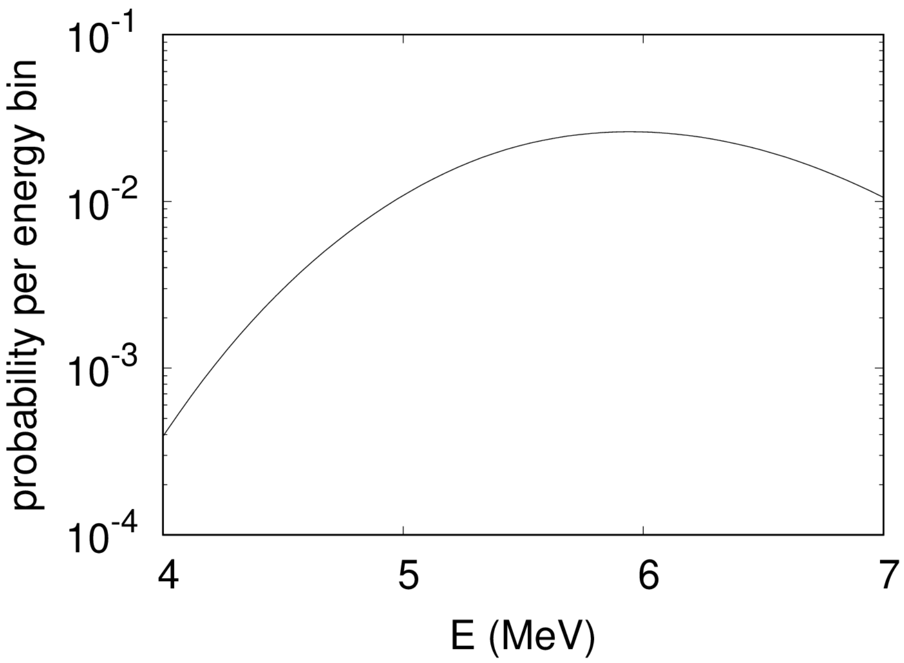
	CAPTURED	NON CAPTURED
CF	● ●	
ICF	●	●

- Act with $\left(P_1+Q_1\right)\left(P_2+Q_2\right)=1$ on the wave function:

$$\tilde{\Psi}\left(x_1,x_2,t\right)=\left(P_1P_2+P_1Q_2+Q_1P_2+Q_1Q_2\right)\tilde{\Psi}\left(x_1,x_2,t\right)=\underline{\Psi_{CF}}+\underline{\Psi_{ICF}}+\underline{\Psi_{SCATT}}$$

Energy Projection of the Wave Function

Schafer & Kulander,
PRA **42** (1990) 5794



♦ **Energy spectra** of $\Psi(t)$ as expectation values of **the window operator**

$$\hat{\boldsymbol{\Delta}}(E_k, n, \epsilon) \equiv \frac{\epsilon^{2^n}}{(\hat{\mathcal{H}} - E_k)^{2^n} + \epsilon^{2^n}}$$

$$E_{k+1} = E_k + 2\epsilon$$

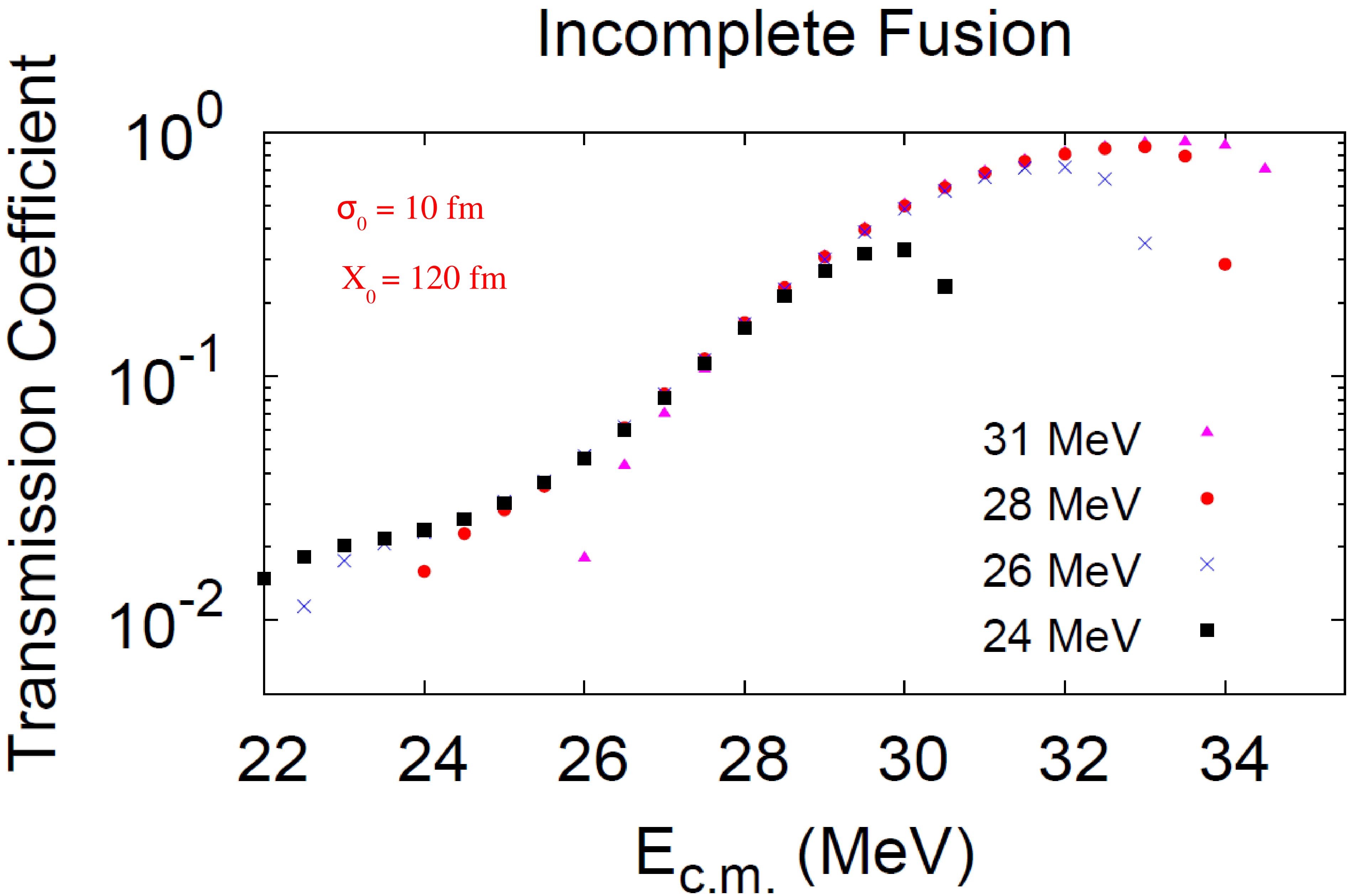
♦ $\mathcal{P}(E_k) = \langle \Psi | \hat{\Delta} | \Psi \rangle$, for instance, **n = 2** :

$$(\hat{H} - E_k + \sqrt{i}\,\epsilon)(\hat{H} - E_k - \sqrt{i}\,\epsilon)\,|\chi_k\rangle = \epsilon^2\,|\Psi\rangle$$

→

$$\mathcal{P}(E_k) = \langle \chi_k | \chi_k \rangle$$

Results



Incomplete Fusion
Transmission Coefficient
10^0
10^{-1}
10^{-2}
σ_0 = 10 fm
X_0 = 120 fm
31 MeV
28 MeV
26 MeV
24 MeV
22
24
26
28
30
32
34
$E_{c.m.}$ (MeV)

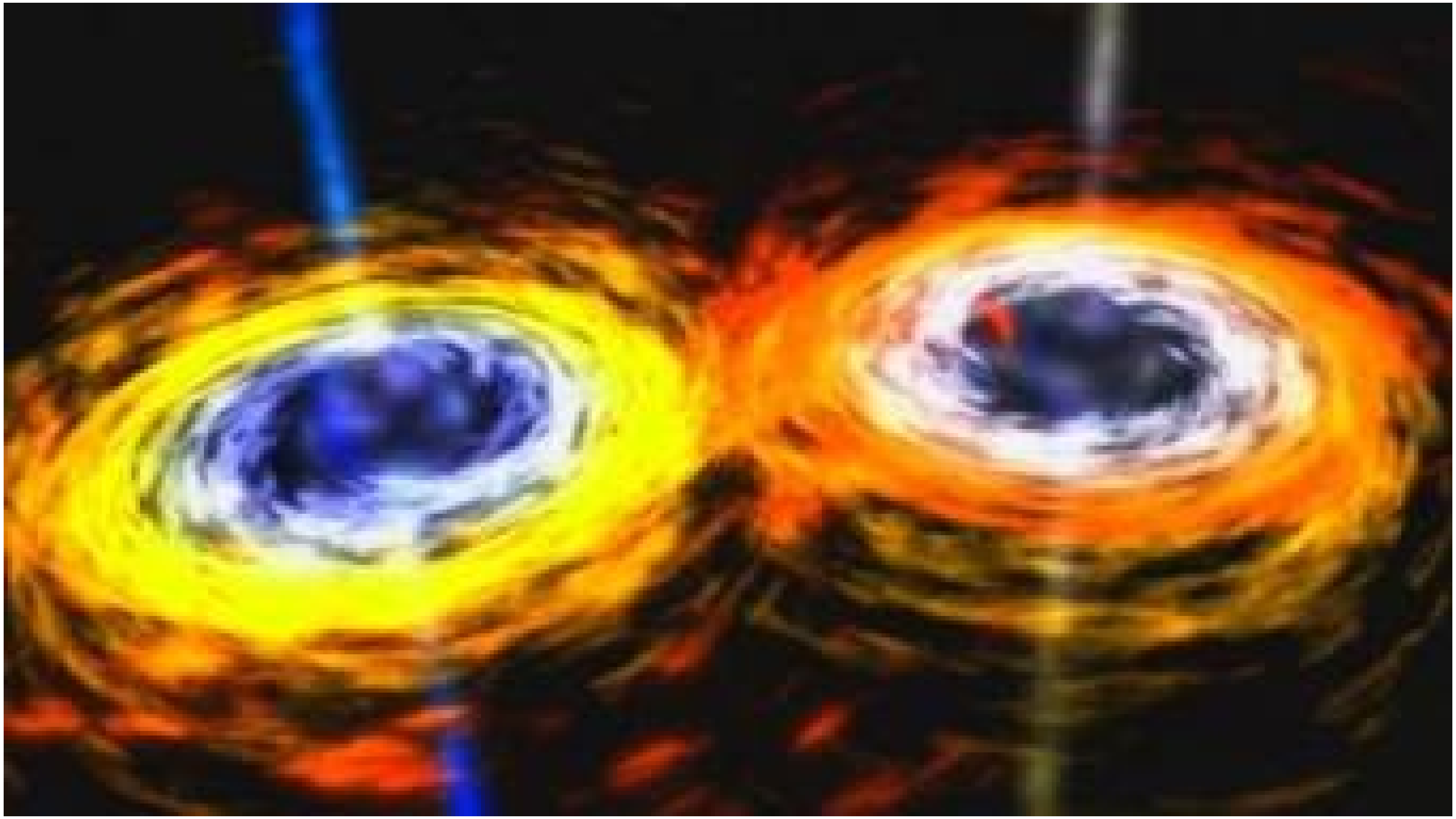
Summarising

Boselli & AD-T, Physical Review C **92** (2015) 044610

- **Wave-packet dynamics** is a useful tool for modelling low-energy fusion dynamics of heavy ions and weakly bound nuclei.
- **Complete & incomplete fusion** can unambiguously be separated in the **configuration space**.
- **A three-dimensional quantum dynamical model** using wave-packet dynamics is being developed.

Lecture 2

How do two ^{12}C nuclei fuse at sub-barrier energies?



Picture taken from BBC News

AD-T & Wiescher, Physical Review C **97** (2018) 055802

Fusion Cross Section & Astrophysical S-Factor

$$S(E) = \sigma(E)\, E\, \exp(2\pi\eta)$$

Structural factor [MeV barn]

Fusion cross section [barn = 10^{-28} m^2]

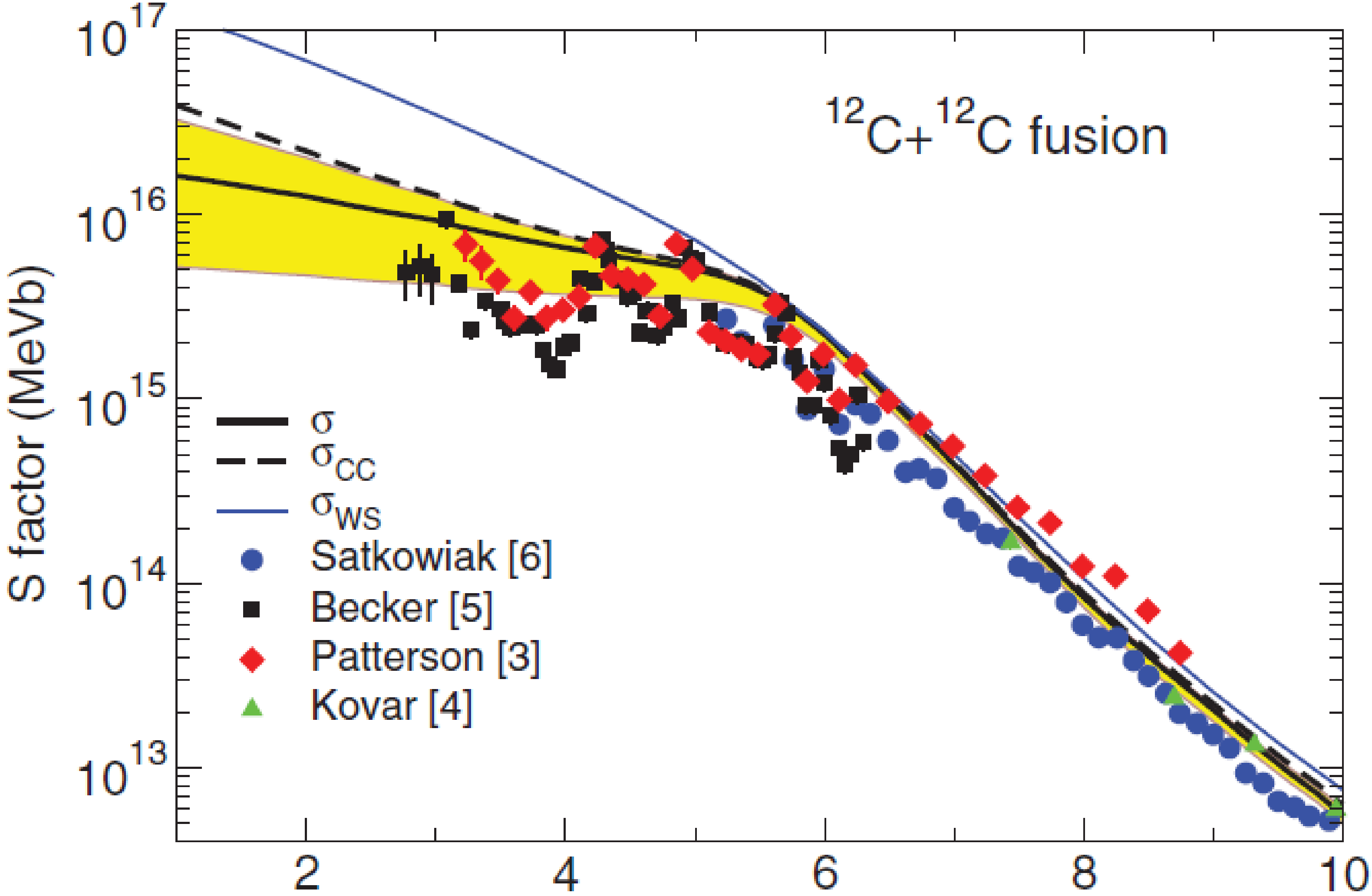
$$\eta = \left(\frac{\mu}{2}\right)^{1/2} \frac{Z_1 Z_2 e^2}{\hbar E^{1/2}}$$

Sommerfeld parameter

S(E) represents the fusion cross section free of Coulomb suppression, which is adequate for extrapolation towards stellar energies

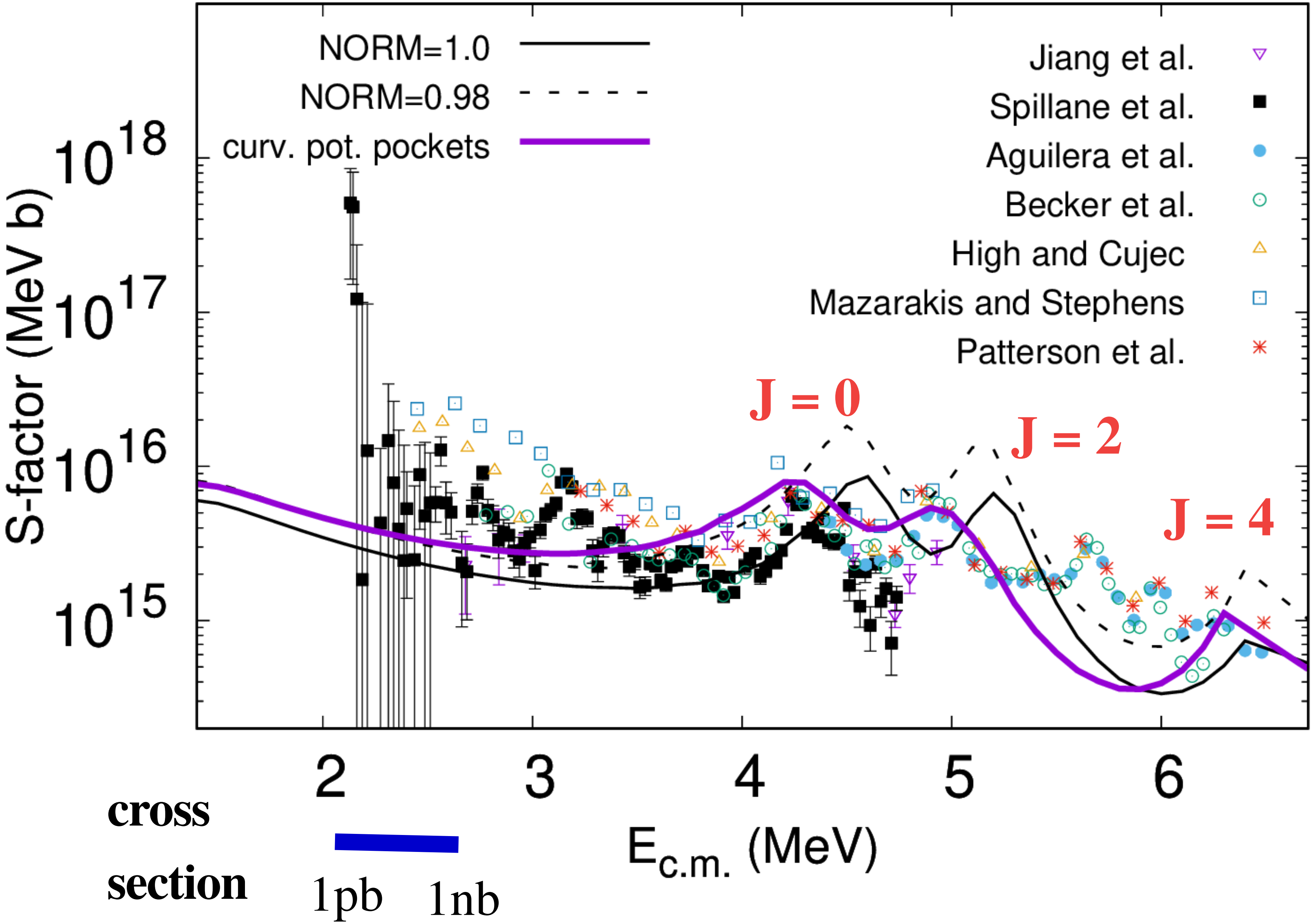
Coupled-Channels Calculations for $^{12}C + ^{12}C$

Jiang, Esbensen et al., PRL **110** (2013) 072701



Ec.m. (MeV)

Astrophysical S-Factor for $^{12}C + ^{12}C$ Fusion



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D.J. Tannor, Quantum Mechanics: a Time-Dependent Perspective, USB, 2007

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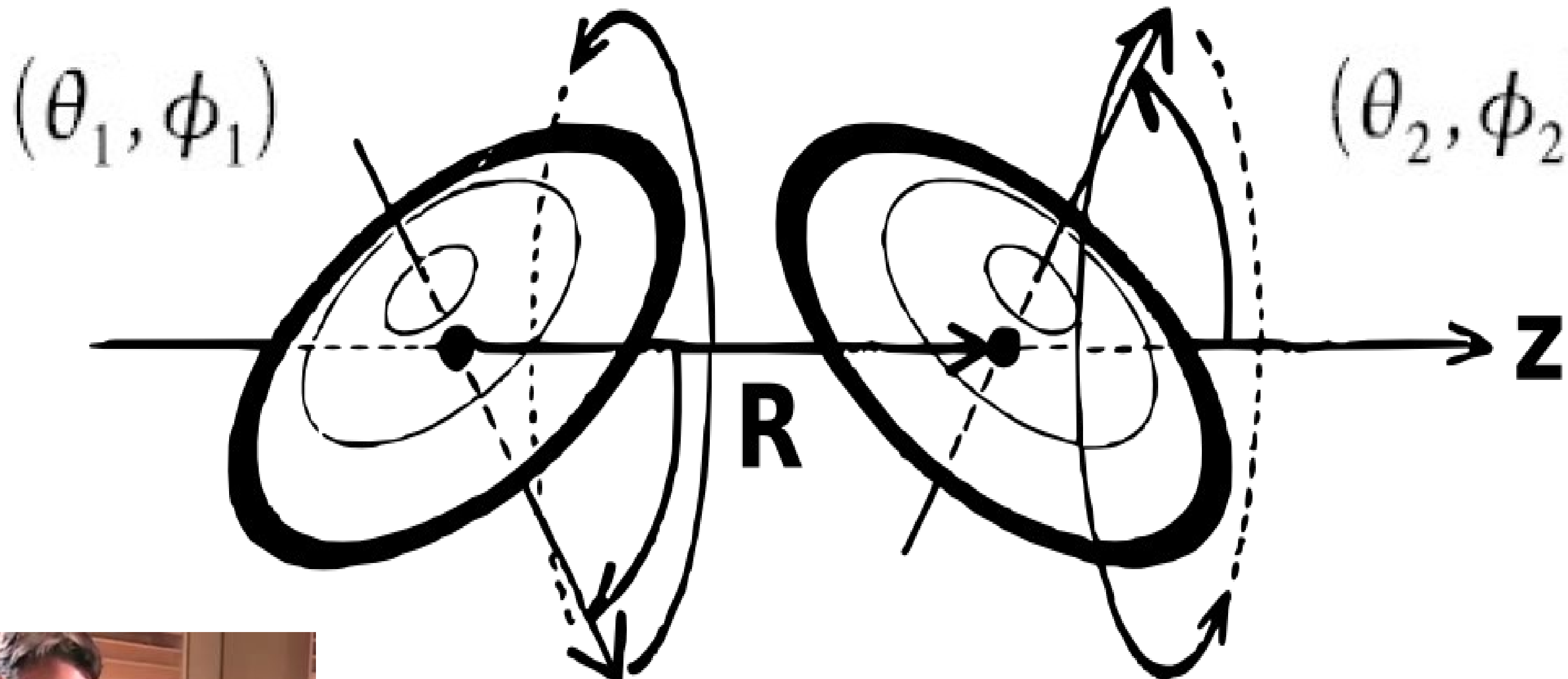
  $\hat{H}$ is the model Hamiltonian

- **Analysis:** extraction of probabilities from the time-dependent wave function

The ^{12}C + ^{12}C Molecular Structure

Greiner, Park & Scheid, in Nuclear Molecules, World Scientific, 1994



Quadrupole deformation of ^{12}C: ~ **- 0.5**

How does this molecular structure affect low-energy fusion?

Initial state $\Psi(t=0)$: the ^{12}C nuclei are well separated

$$\Psi_0(R,\theta_1,k_1,\theta_2,k_2) = \chi_0(R)\,\psi_0(\theta_1,k_1,\theta_2,k_2),$$

Radial motion (→ $\chi_0(R)$)

Internal rotational motion (→ ψ_0)

$$\chi_0(R) = (\sqrt{\pi}\sigma)^{-1/2}\ \exp\Big[-\frac{(R-R_0)^2}{2\sigma^2}\Big]\, e^{iP_0(R-R_0)},$$

$$\begin{aligned}\psi_0(\theta_1,k_1,\theta_2,k_2) = {} & \big[\zeta_{j_1,m_1}(\theta_1,k_1)\zeta_{j_2,m_2}(\theta_2,k_2) \\ & +(-1)^J\zeta_{j_2,-m_2}(\theta_1,k_1)\zeta_{j_1,-m_1}(\theta_2,k_2)\big] \\ & /\sqrt{2+2\,\delta_{j_1,j_2}\delta_{m_1,-m_2}},\end{aligned}$$

where $\zeta_{j,m}(\theta,k) = \sqrt{\frac{(2j+1)(j-m)!}{2\,(j+m)!}}\,P_j^m(\cos\theta)\,\delta_{km}$,
and P_j^m are associated Legendre functions.

Kinetic-Energy of Two Deformed Colliding Nuclei

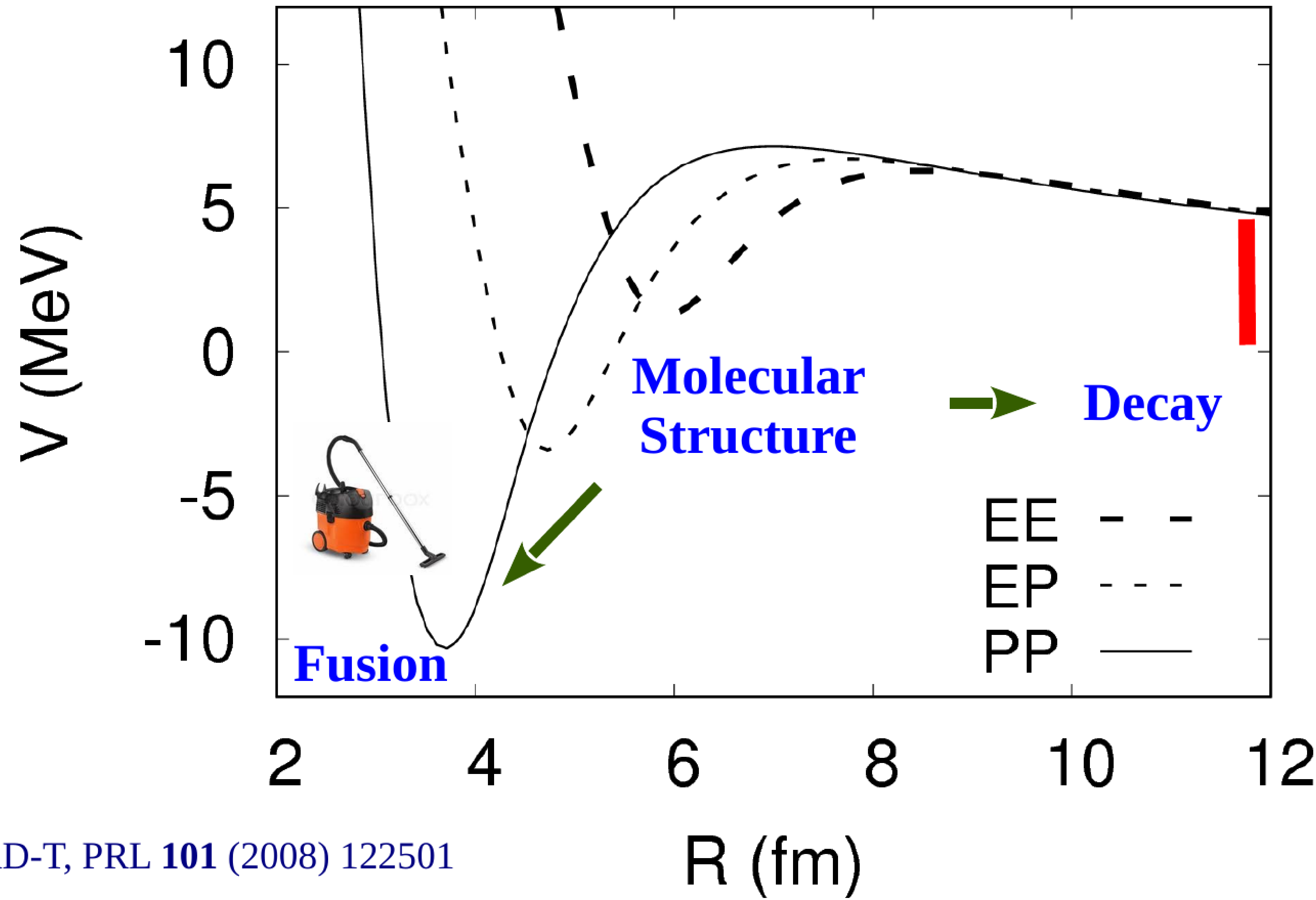
Gatti *et al.*, JCP **123** (2005) 174311

$$\frac{2\hat{T}}{\hbar^2} = -\frac{1}{\mu}\frac{\partial^2}{\partial R^2} + \left(\frac{1}{I_1} + \frac{1}{\mu R^2}\right)\hat{j}_1^2 + \left(\frac{1}{I_2} + \frac{1}{\mu R^2}\right)\hat{j}_2^2$$
$$+ \frac{1}{\mu R^2}\left[\hat{j}_{1,+}\hat{j}_{2,-} + \hat{j}_{1,-}\hat{j}_{2,+} + J(J+1)\right.$$
$$\left. - 2k_1^2 - 2k_1 k_2 - 2k_2^2\right] - \frac{C_+(J,K)}{\mu R^2}(\hat{j}_{1,+} + \hat{j}_{2,+})$$
$$- \frac{C_-(J,K)}{\mu R^2}(\hat{j}_{1,-} + \hat{j}_{2,-})$$

Coriolis interaction

μ is the reduced mass for the radial motion,
I_i is the ^{12}C rotational inertia,
J is the total angular momentum with projection $K = k_1 + k_2$,
$C_\pm(J,K) = \sqrt{J(J+1) - K(K\pm 1)}$,
$\hat{j}_i^2 = -\frac{1}{\sin\theta_i}\frac{\partial}{\partial\theta_i}\sin\theta_i\frac{\partial}{\partial\theta_i} + \frac{k_i^2}{\sin^2\theta_i}$,
$\hat{j}_{i,\pm} = \pm\frac{\partial}{\partial\theta_i} - k_i\cot\theta_i$, with $k_i \to k_i \pm 1$.

Collective Potential-Energy Landscape for $^{12}C + ^{12}C$

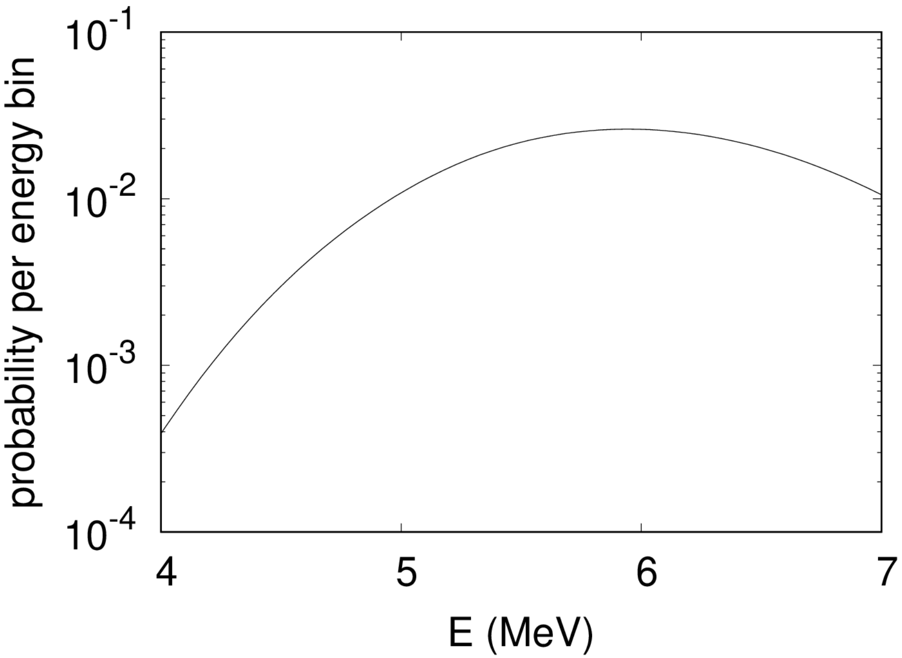


AD-T, PRL **101** (2008) 122501

Moeller & Iwamoto, NPA **575** (1994) 381

Energy Projection of the Wave Function

Schafer & Kulander,
PRA **42** (1990) 5794



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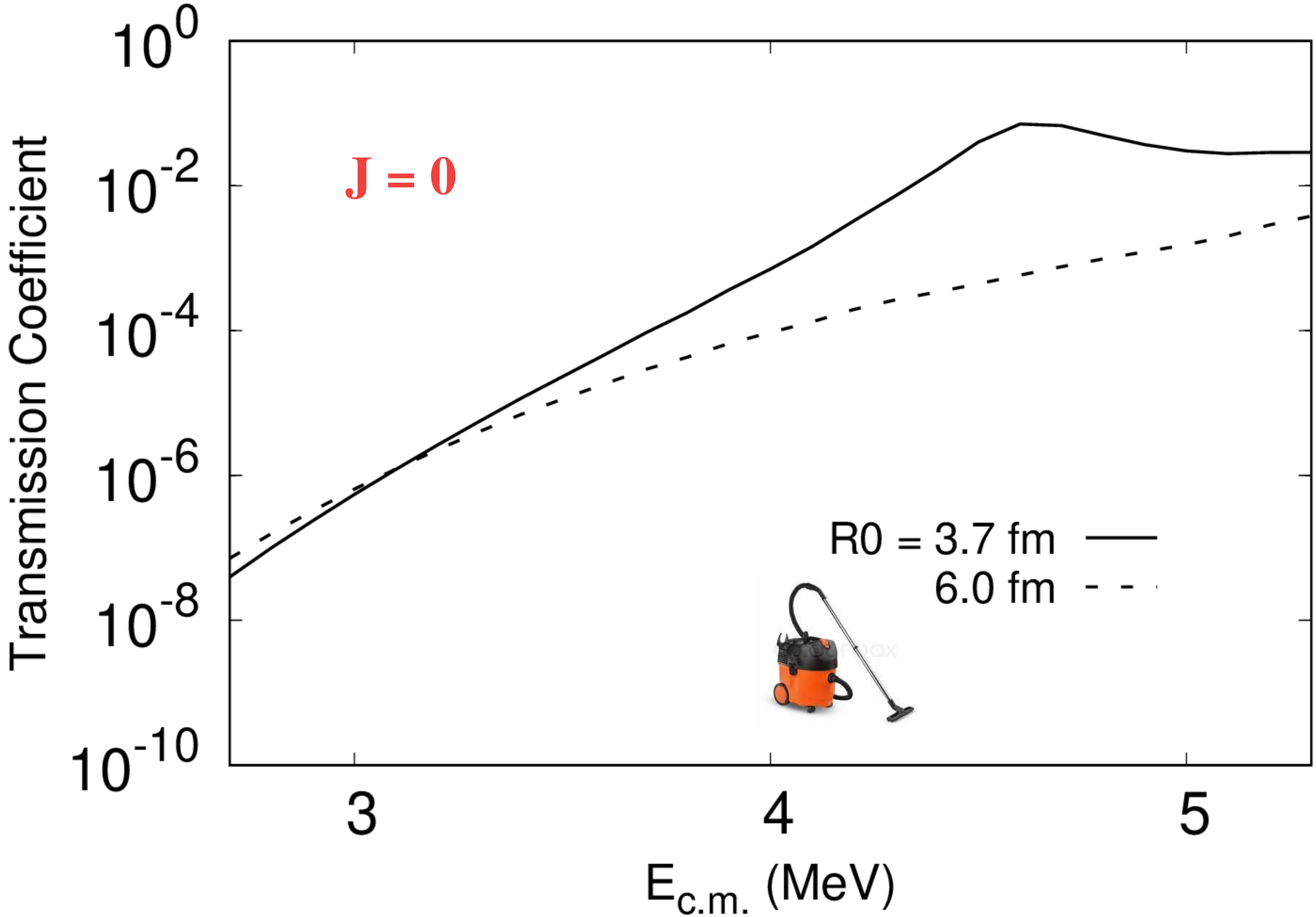
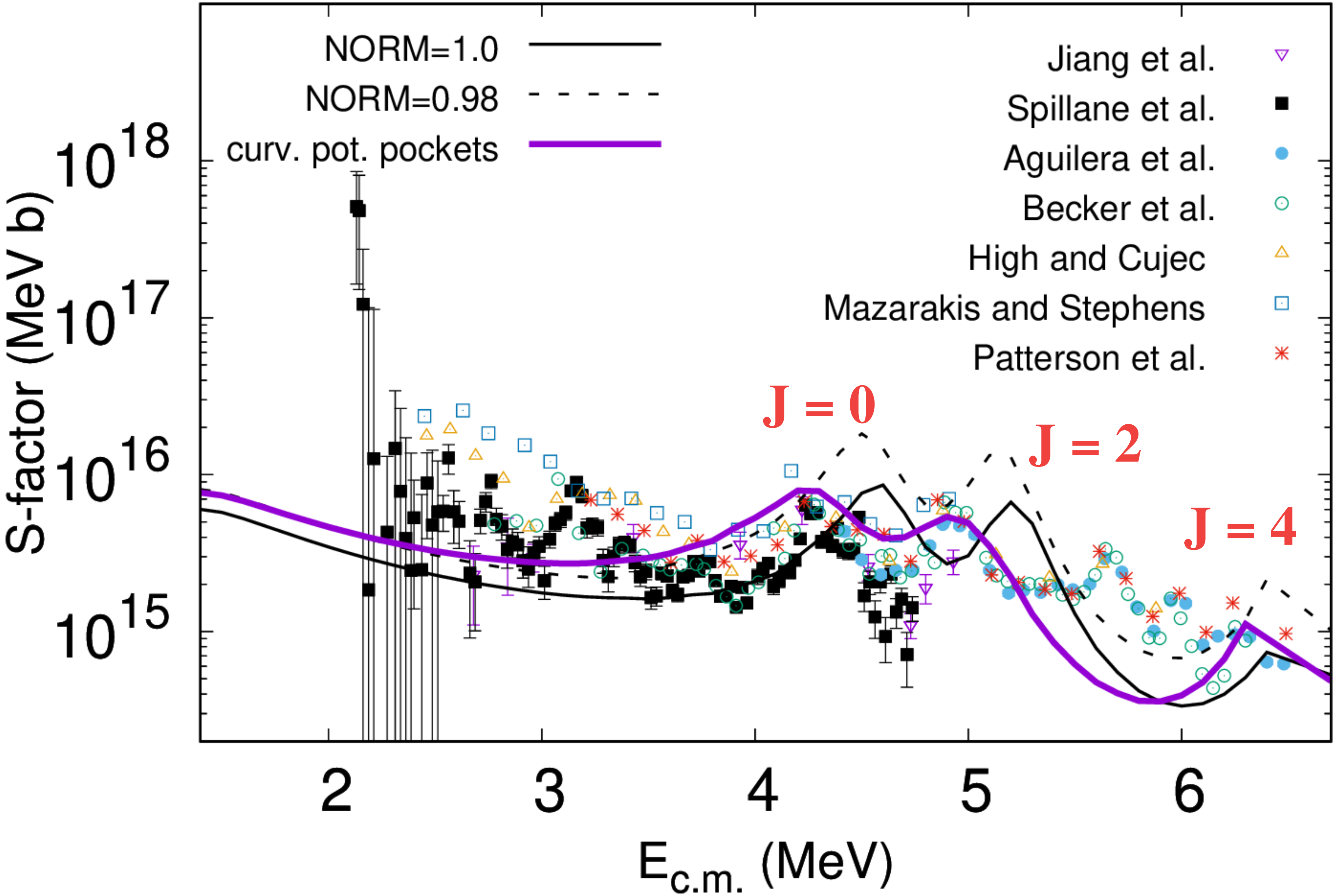
$$\mathcal{P}(E_k) = \langle \chi_k | \chi_k \rangle$$

Role of the imaginary fusion potential in the transmission coefficient



Astrophysical S-Factor for $^{12}C + ^{12}C$ Fusion

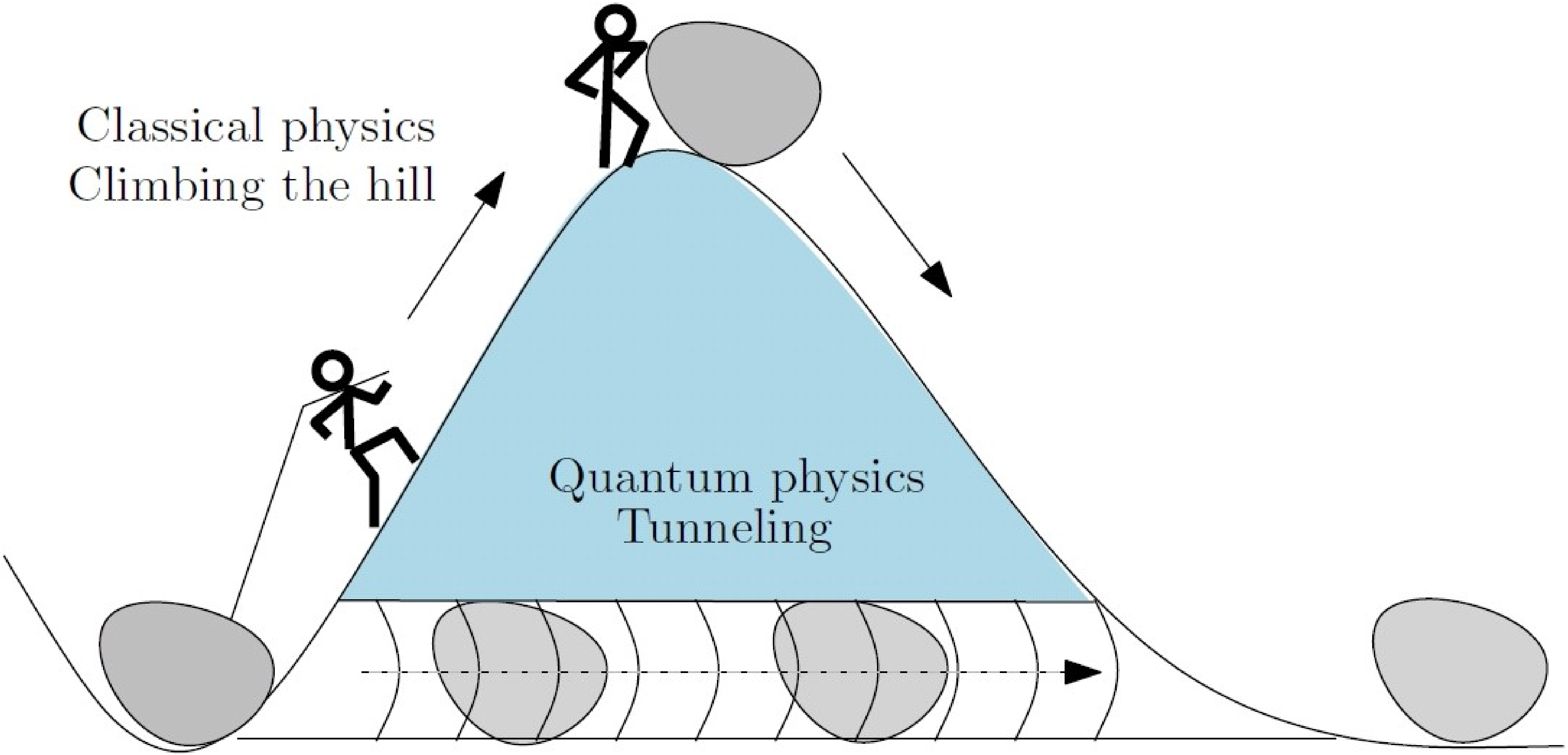


- The **fusion imaginary potential** for *specific* dinuclear configurations is crucial for the appearance of resonances.
- **Three resonant structures** are revealed in the calculations, reproducing similar structures in the experimental data.
- **Resonant structures** in the experimental data that are not explained may be due to **cluster effects** in the nuclear molecule.

AD-T & Wiescher, Physical Review C 97 (2018) 055802

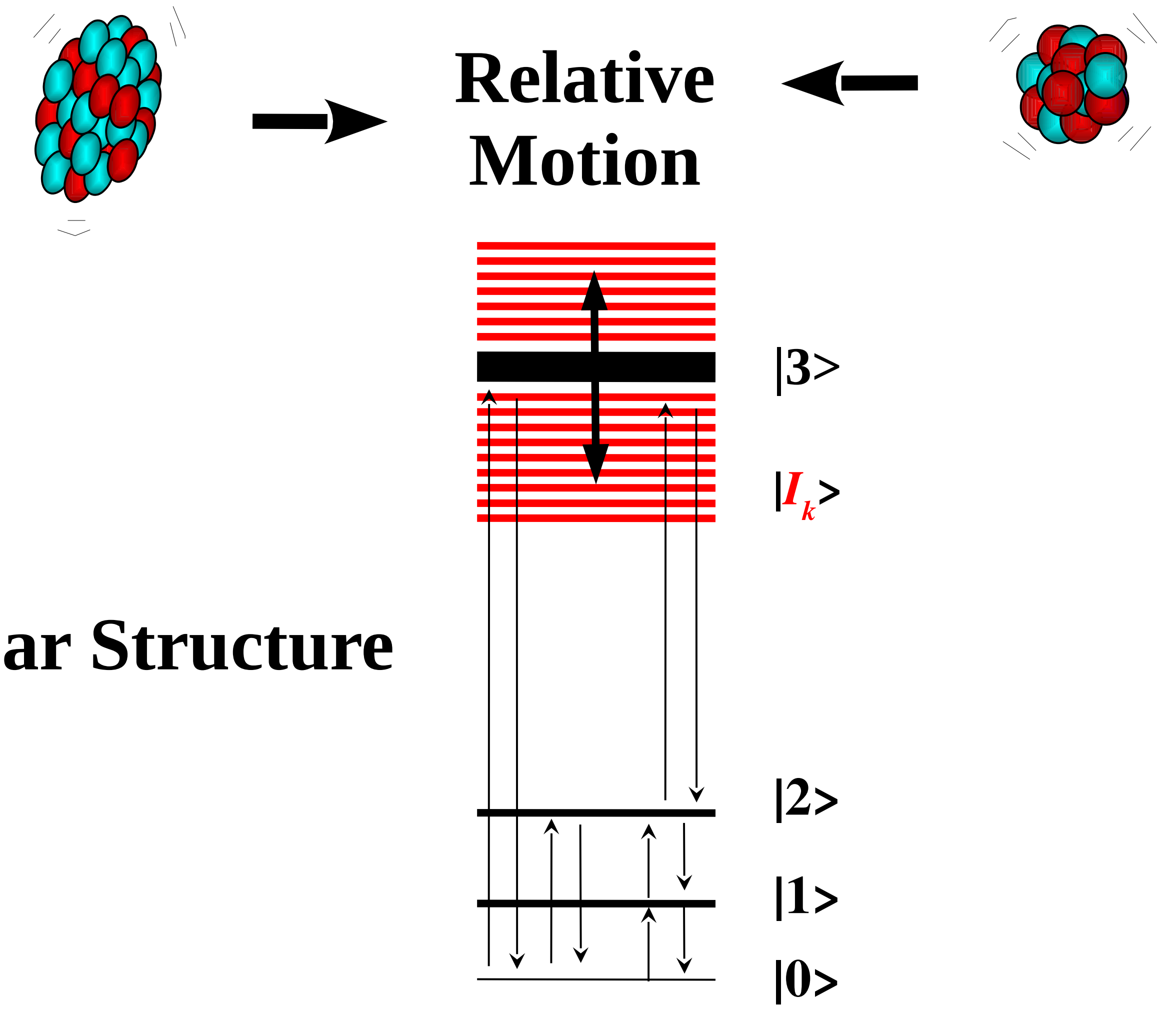
Lecture 3

Quantum Tunneling in Nuclear Fusion



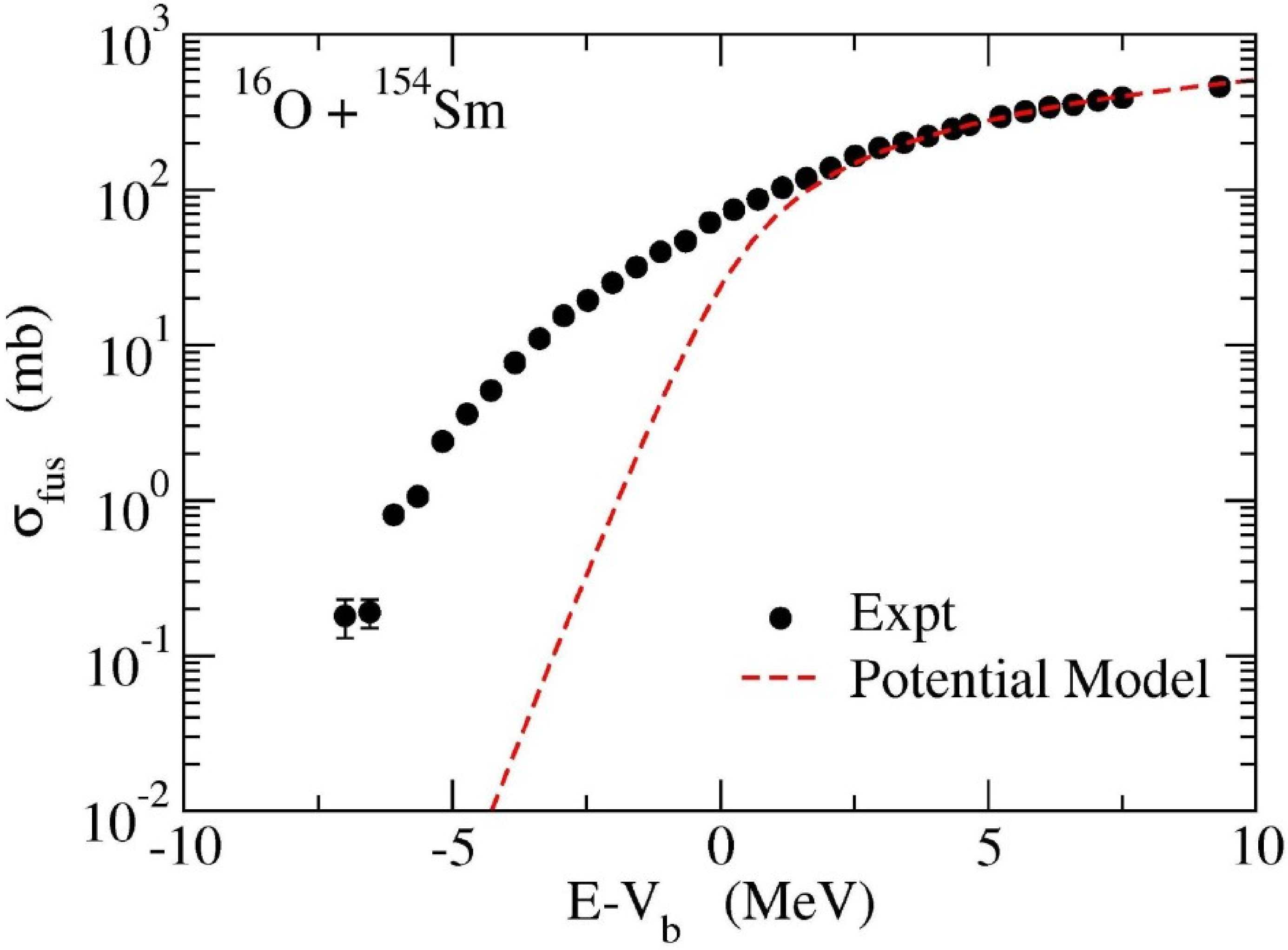
Vockerodt & A D-T, Physical Review C **100** (2019) 034606

Fusion of Complex Atomic Nuclei



Nuclear Structure

Interplay between **nuclear structure** and **relative motion** determines **fusion cross sections**



Potential model:
Reproduces the data reasonably well for $E > V_b$

Underpredicts σ_{fus} for $E < V_b$

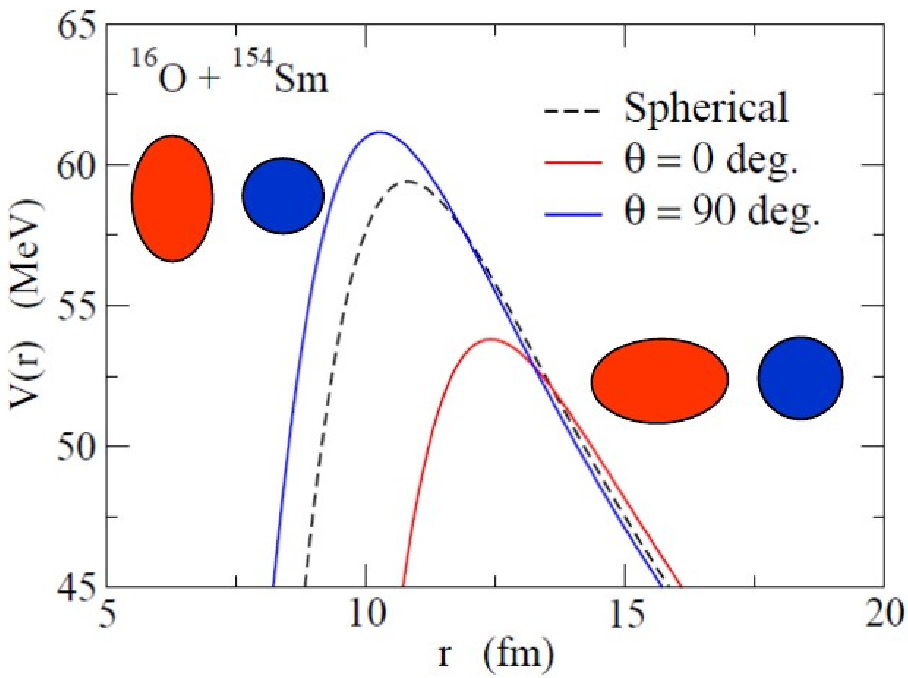
cf. seminal work:
R.G. Stokstad et al., PRL41('78)465
PRC21('80)2427

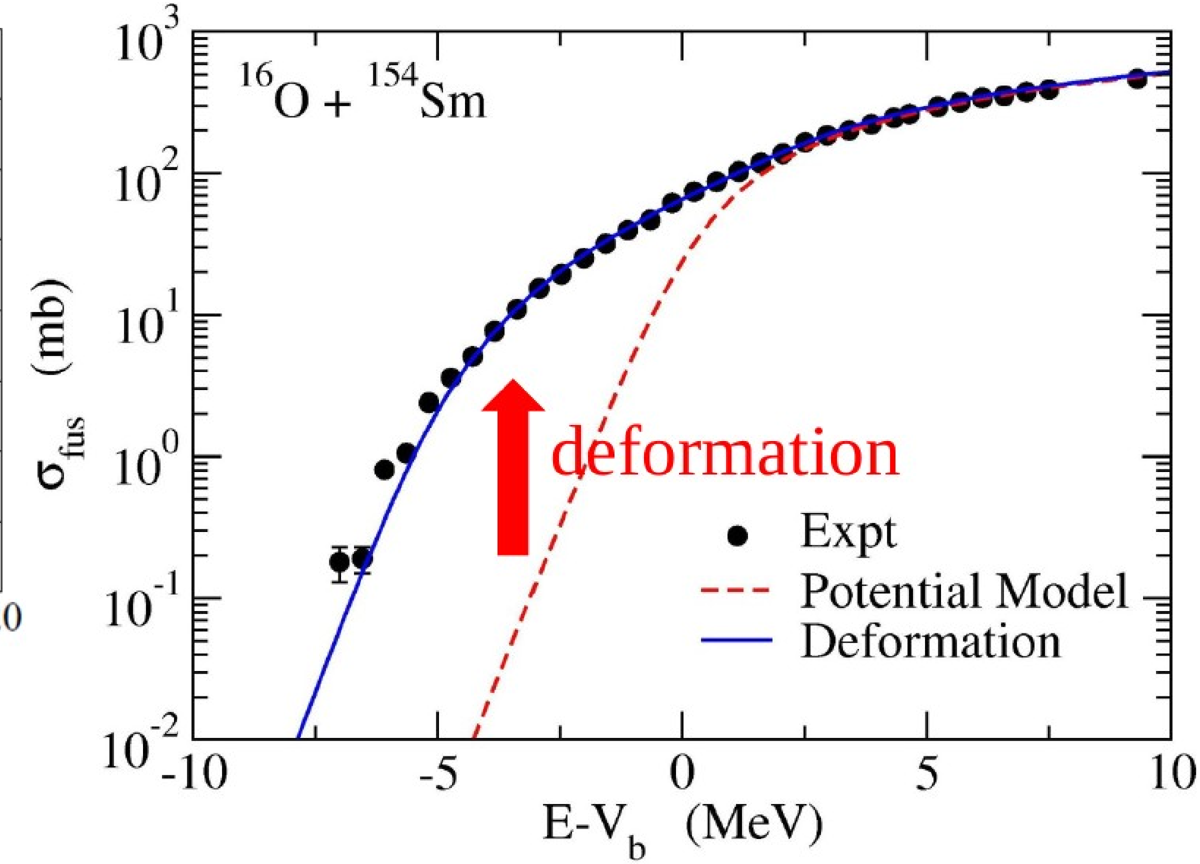
Courtesy of K. Hagino

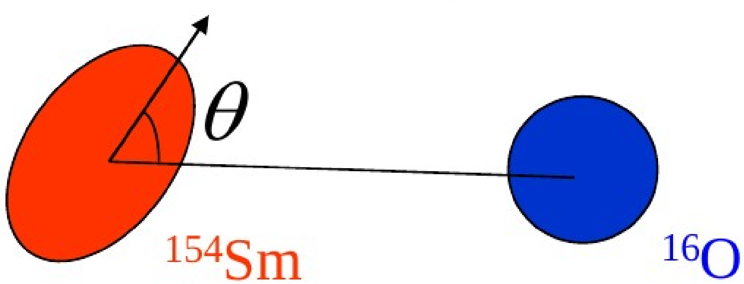
Effect of nuclear deformation

Courtesy of K. Hagino

^{154}Sm : a deformed nucleus with $\beta_2 \sim 0.3$

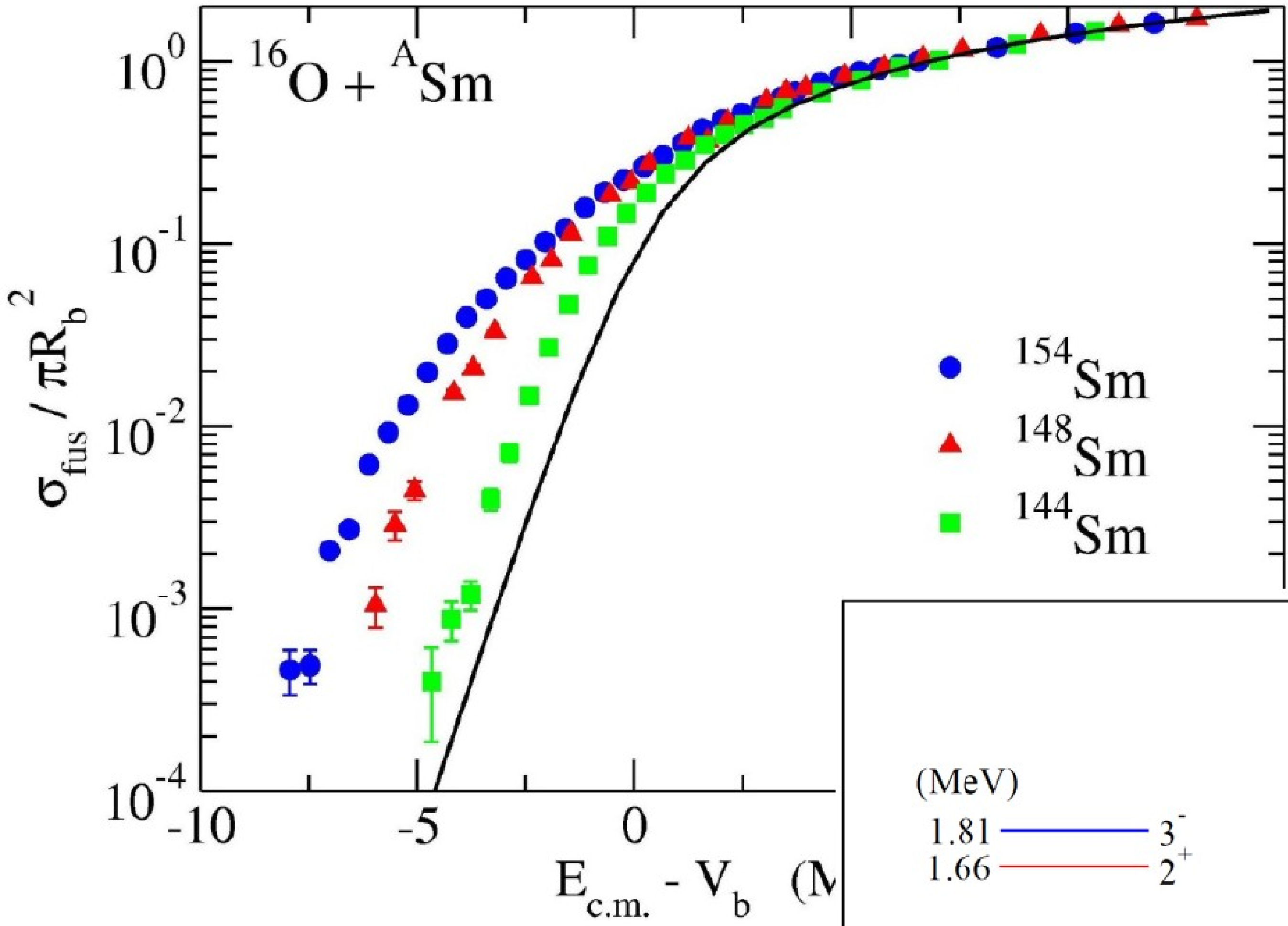


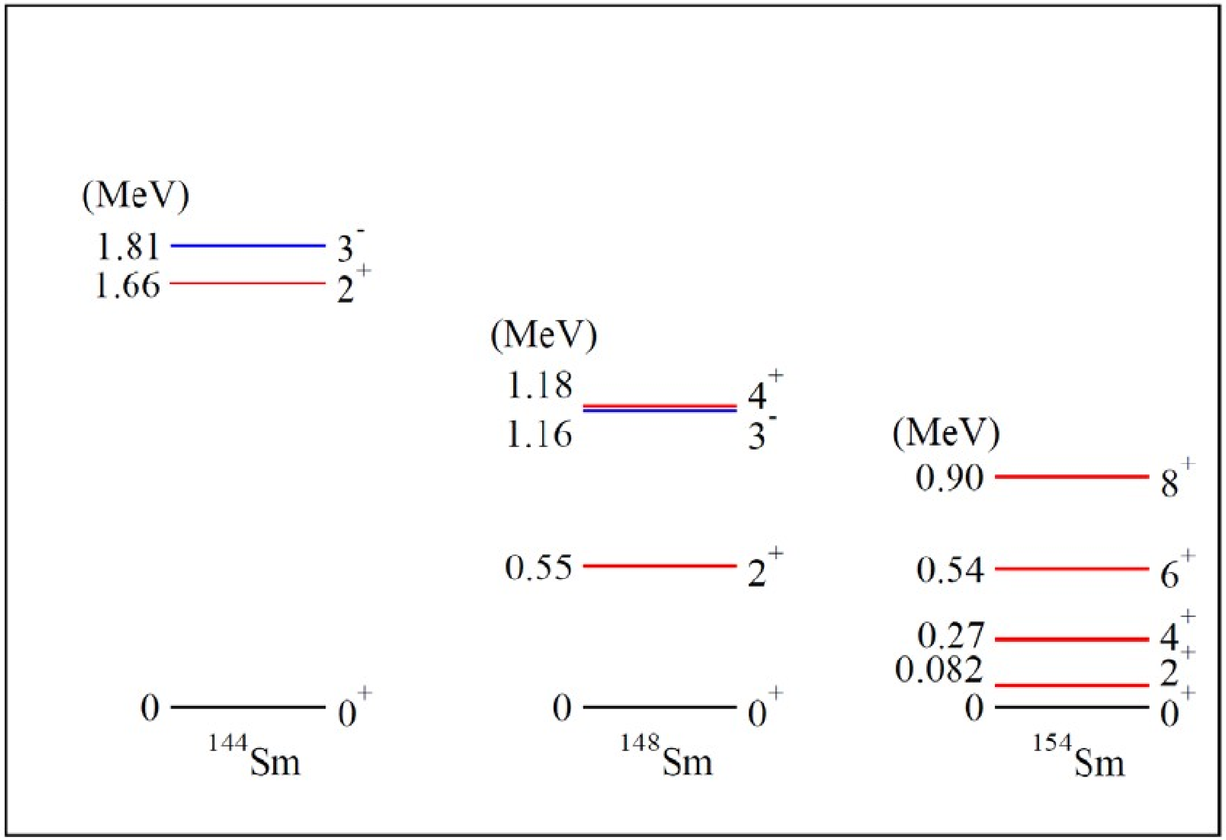




$$\sigma_{\text{fus}}(E) = \int_0^1 d(\cos\theta)\sigma_{\text{fus}}(E;\theta)$$

Fusion: strong interplay between nuclear structure and nuclear reaction





Strong target dependence
at $E < V_b$

Courtesy of K. Hagino

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D.J. Tannor, Quantum Mechanics: a Time-Dependent Perspective, USB, 2007

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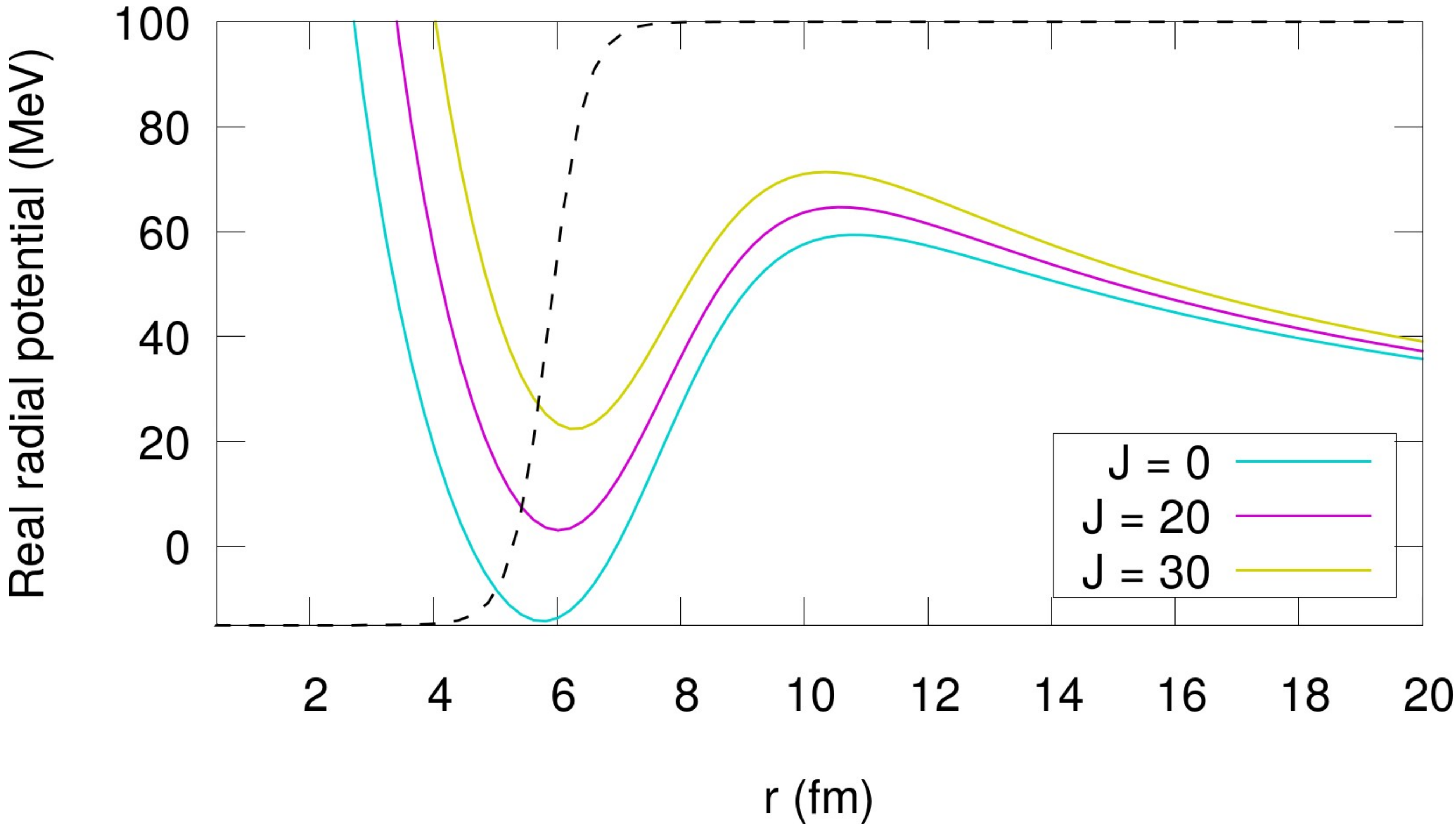
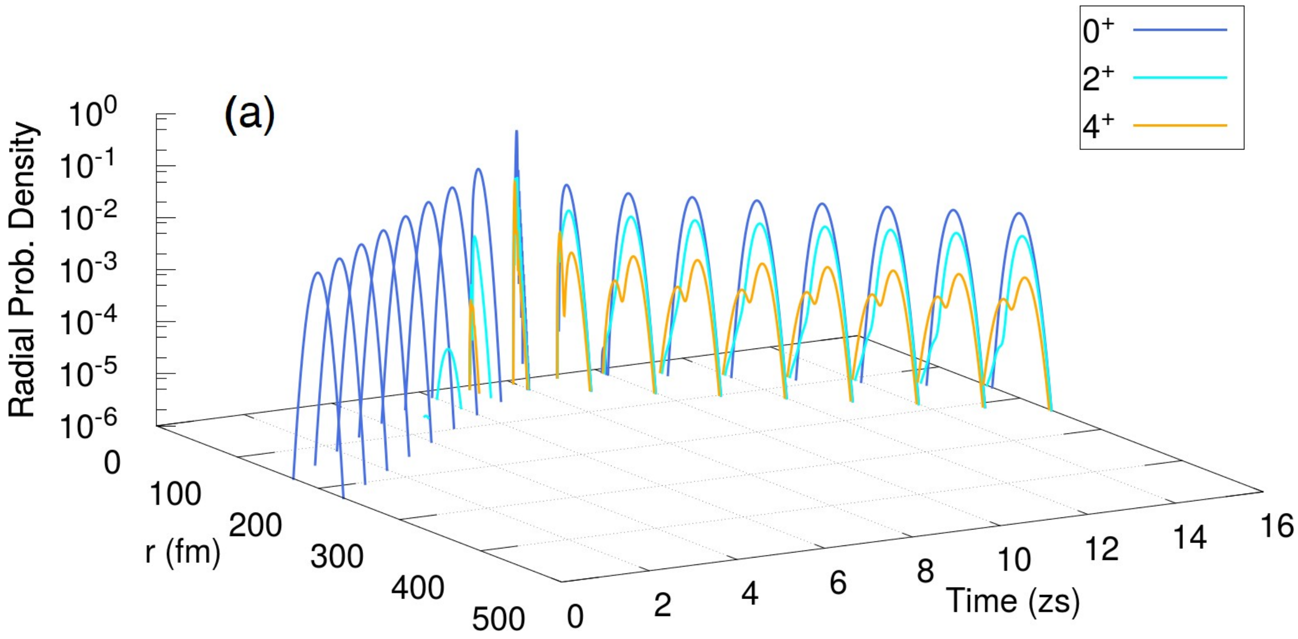
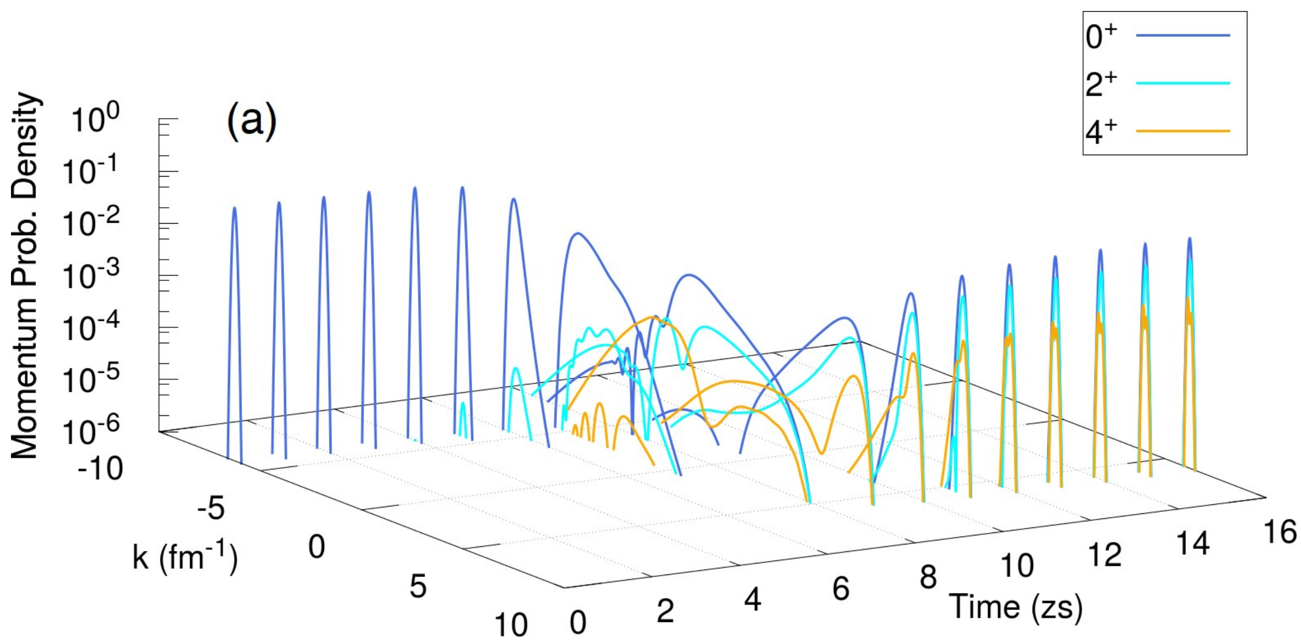
Interaction Potentials for ^{16}O + ^{154}Sm



Vockerodt & A D-T, PRC 100 (2019) 034606

Coupled-Channel Wave-Packet Dynamics for $^{16}O + {}^{154}Sm$





Vockerodt & A D-T, PRC 100 (2019) 034606

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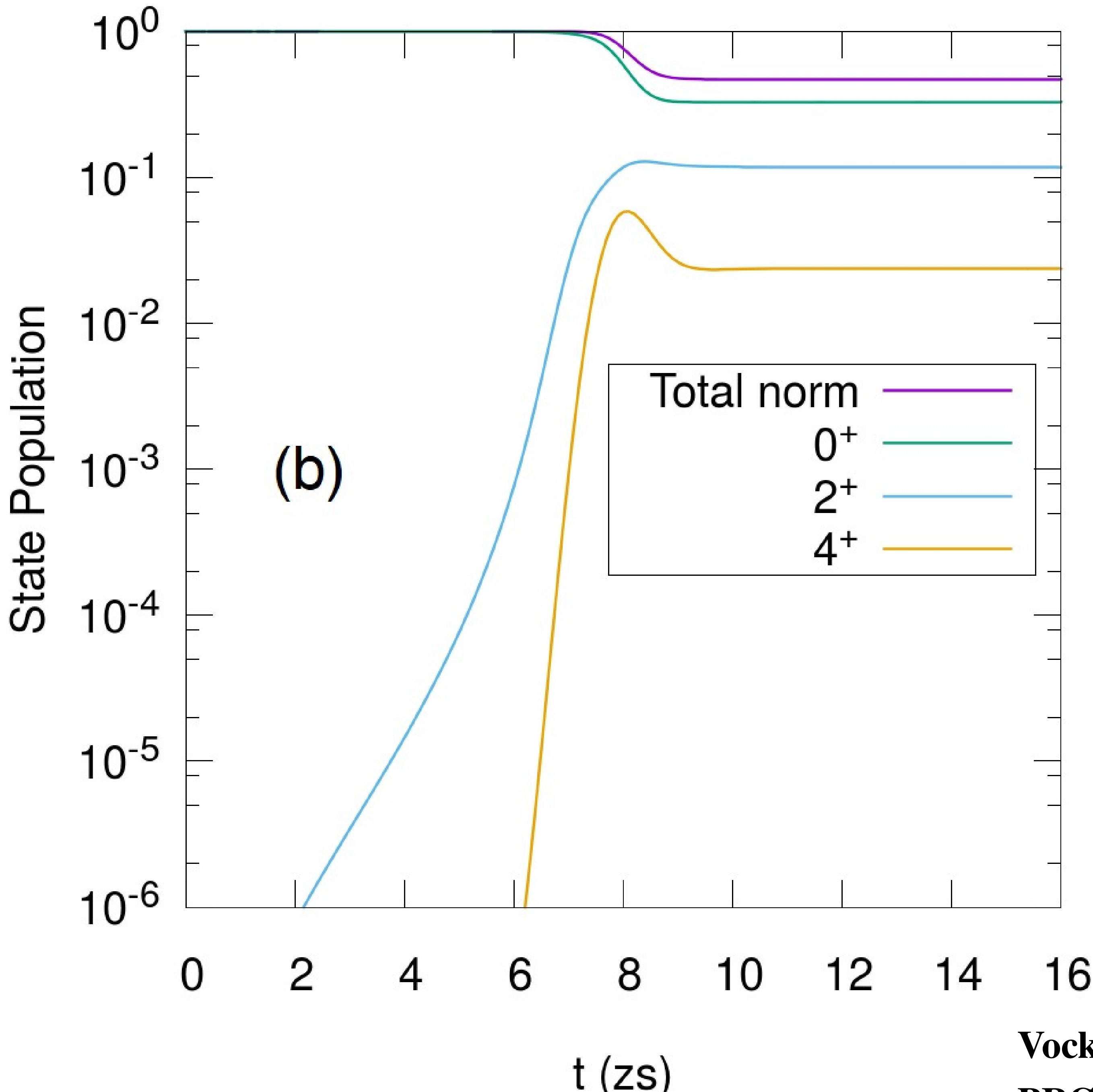

Vockerodt & A D-T,
PRC 100 (2019) 034606

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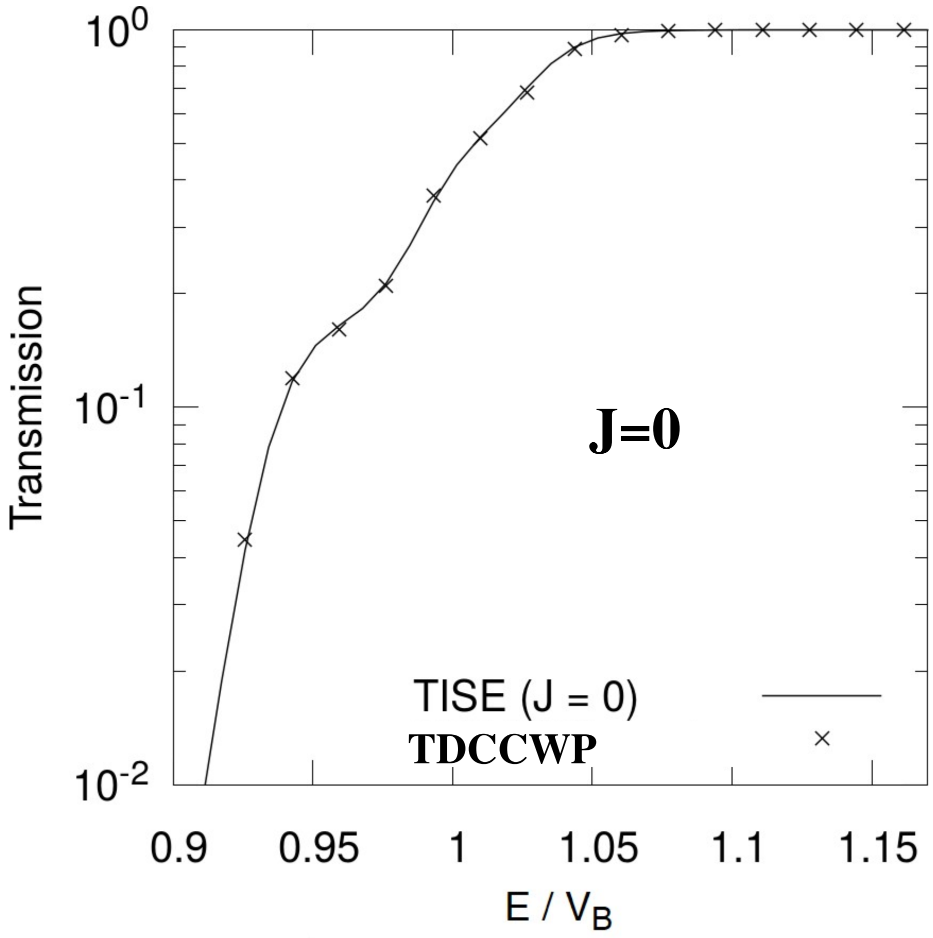

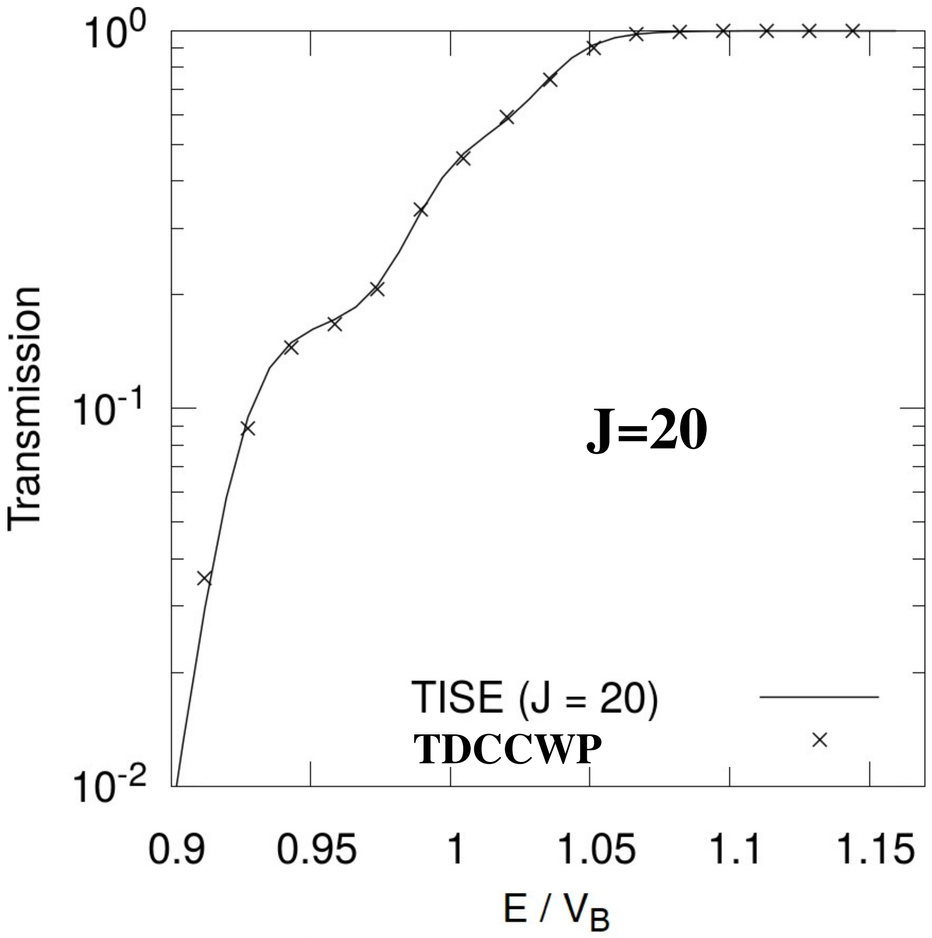

Vockerodt & A D-T, PRC 100 (2019) 034606

Computer Physics Communications 123 (1999) 143–152

Computer Physics Communications

www.elsevier.nl/locate/cpc

CCFULL

A program for coupled-channel calculations with all order couplings for heavy-ion fusion reactions

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Received 6 April 1999

Abstract

A FORTRAN 77 program that calculates fusion cross sections and mean angular momenta of the compound nucleus under the influence of couplings between the relative motion and several nuclear collective motions is presented. The no-Coriolis approximation is employed to reduce the dimension of coupled-channel equations. The program takes into account the effects of nonlinear couplings to all orders, which have been shown to play an important role in heavy-ion fusion reactions at subbarrier energies.

PACS: 25.70.Jj; 24.10.Eq

Keywords: Heavy-ion subbarrier fusion reactions; Coupled-channel equations; Higher order coupling; No-Coriolis approximation; Incoming wave boundary condition; Fusion cross section; Mean angular momentum; Spin distribution; Fusion barrier distribution; Multi-dimensional quantum tunneling

2. Coupled-channel equations

For heavy-ion fusion reactions, to a good approximation one can replace the angular momentum of the relative motion in each channel by the total angular momentum J [8,9]. This approximation, often referred to as no-Coriolis approximation or isocentrifugal approximation, is used in the program. The coupled-channel equations then read

$$\left[-\frac{\hbar^2}{2\mu}\frac{d^2}{dr^2}+\frac{J(J+1)\hbar^2}{2\mu r^2}+V_N^{(0)}(r)+\frac{Z_P Z_T e^2}{r}+\epsilon_n-E\right]\psi_n(r)+\sum_m V_{nm}(r)\psi_m(r)=0, \tag{1}$$

$$V_N^{(0)}(r)=-\frac{V_0}{1+\exp((r-R_0)/a)}, \qquad R_0=r_0(A_P^{1/3}+A_T^{1/3}),$$

$$\psi_n(r)\to T_n\exp\left(-i\int_{r_{\min}}^{r}k_n(r')\,dr'\right), \quad r\leqslant r_{\min},$$

$$\to H_J^{(-)}(k_n r)\delta_{n,0}+R_n H_J^{(+)}(k_n r), \quad r>r_{\max},$$

where

$$k_n(r)=\sqrt{\frac{2\mu}{\hbar^2}\left(E-\epsilon_n-\frac{J(J+1)\hbar^2}{2\mu r^2}-V_N(r)-\frac{Z_P Z_T e^2}{r}-V_{nn}(r)\right)}$$

Fusion Cross Section and Mean Angular Momentum

For many examples, we are interested only in the inclusive process, where the intrinsic degree of freedom emerges in any final state. Taking a summation over all possible intrinsic states, the inclusive penetrability is given by

$$P_J(E) = \sum_n \frac{k_n(r_{\min})}{k_0} |T_n|^2 . \tag{17}$$

The fusion cross section and the mean angular momentum of compound nucleus are then calculated by

$$\sigma_{\text{fus}}(E) = \sum_J \sigma_J(E) = \frac{\pi}{k_0^2} \sum_J (2J+1) P_J(E), \tag{18}$$

$$\begin{aligned} \langle l \rangle &= \sum_J J \sigma_J(E) \Big/ \sum_J \sigma_J(E) \\ &= \left(\frac{\pi}{k_0^2} \sum_J J(2J+1) P_J(E) \right) \Big/ \left(\frac{\pi}{k_0^2} \sum_J (2J+1) P_J(E) \right), \end{aligned} \tag{19}$$

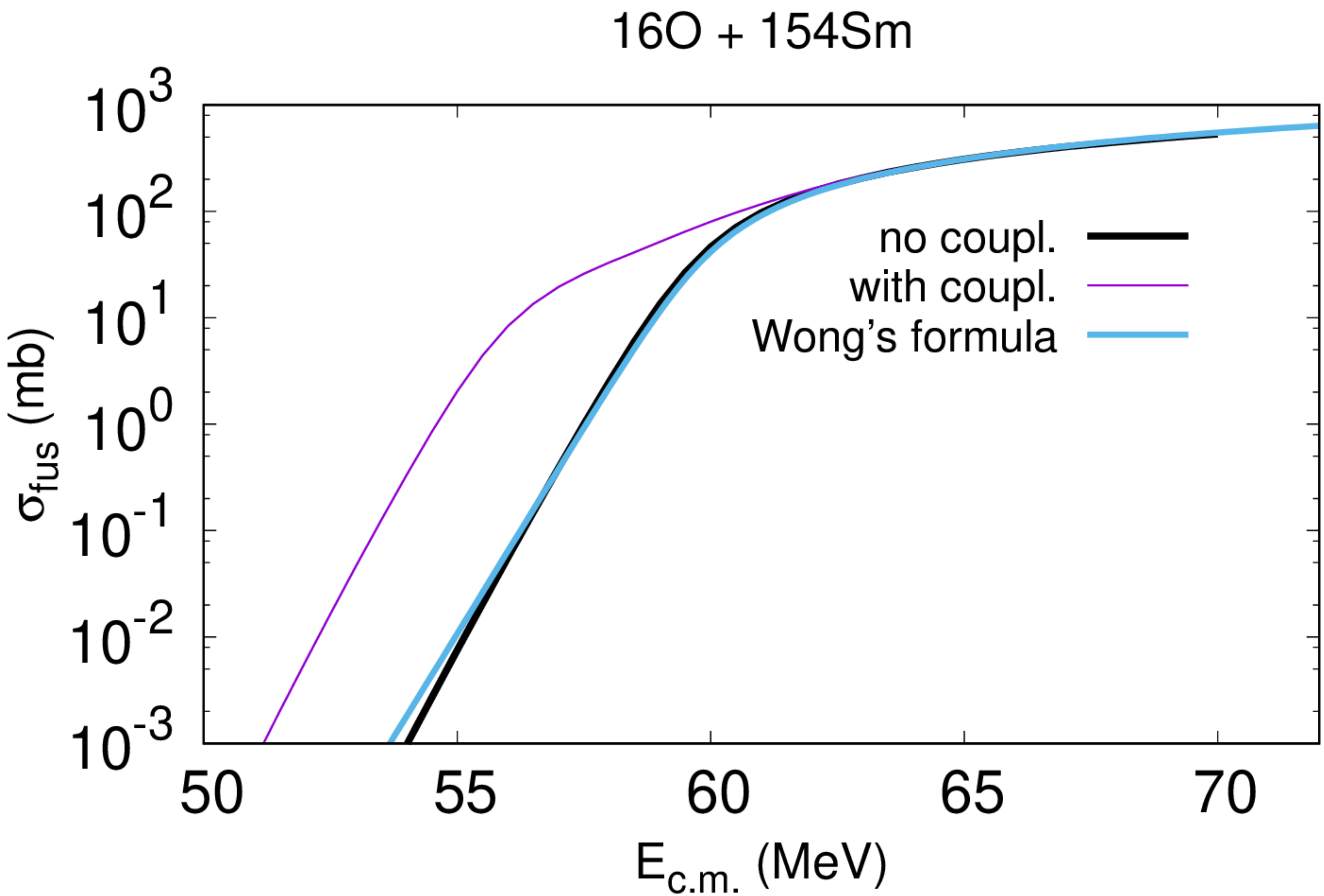
respectively. In the program CCFULL, the summation over the partial wave is truncated at the angular momentum whose contribution to the cross section is less than 10^{-4} times the total cross section.

Input file

```
16.,8.,154.,62.
1.2,-1,1.06,-1
0.082,0.322,0.027,2
1.81,0.205,3,0
6.13,0.733,3,0
0,0.,0.3
165.0,0.95,1.05
50.,70.,0.5
30.,0.05
```

```
The first line:
     AP,ZP,AT,ZT
The second line:
     RP,IVIBROTP,RT,IVIBROTT
          (The radius parameter used in the coupling Hamiltonian)
           (IVIBROT: option for intrinsic degree of freedom
                     = -1; no excitation (inert)
                     = 0 ; vibrational coupling
                     = 1 ; rotational coupling
                IVIBROTP: for projectile excitation
                IVIBROTT: for target excitation)
The third line:
     OMEGAT,BETAT,LAMBDAT,NPHONONT  (if IVIBROTT=0)
     E2T,BETA2T,BETA4T,NROTT        (if IVIBROTT=1)
          (Input for the target excitation)
          (This line is irrelevant if IVIBROTT = -1.)
          (NROT: the number of levels in the rotational band to be
                 included (up to I^pi=2*NROTT+ states are included)
                 e.g. if NROTT=2, then 0+, 2+ and 4+ in the target
                 are included.)
The 4th line:
     OMEGAT2,BETAT2,LAMBDAT2,NPHONONT2
          (Input for target phonon excitation; the second mode of
           excitation.
           For example, the first mode (LAMBDAT) may be a quadrupole
           vib. and the second mode (LAMBDAT2) may be an octupole vib.
           in the target nucleus.)
          (No second target phonon excitation if NPHONONT2=0
                   OMEGAT2, BETAT2, and LAMBDAT2 are irrelevant
                   if NPHONONT2=0)

The 7th line:
     V0,R0,A0
          (Potential parameters)
The 8th line:
     EMIN,EMAX,DE
The 9th line:
     RMAX,DR
```

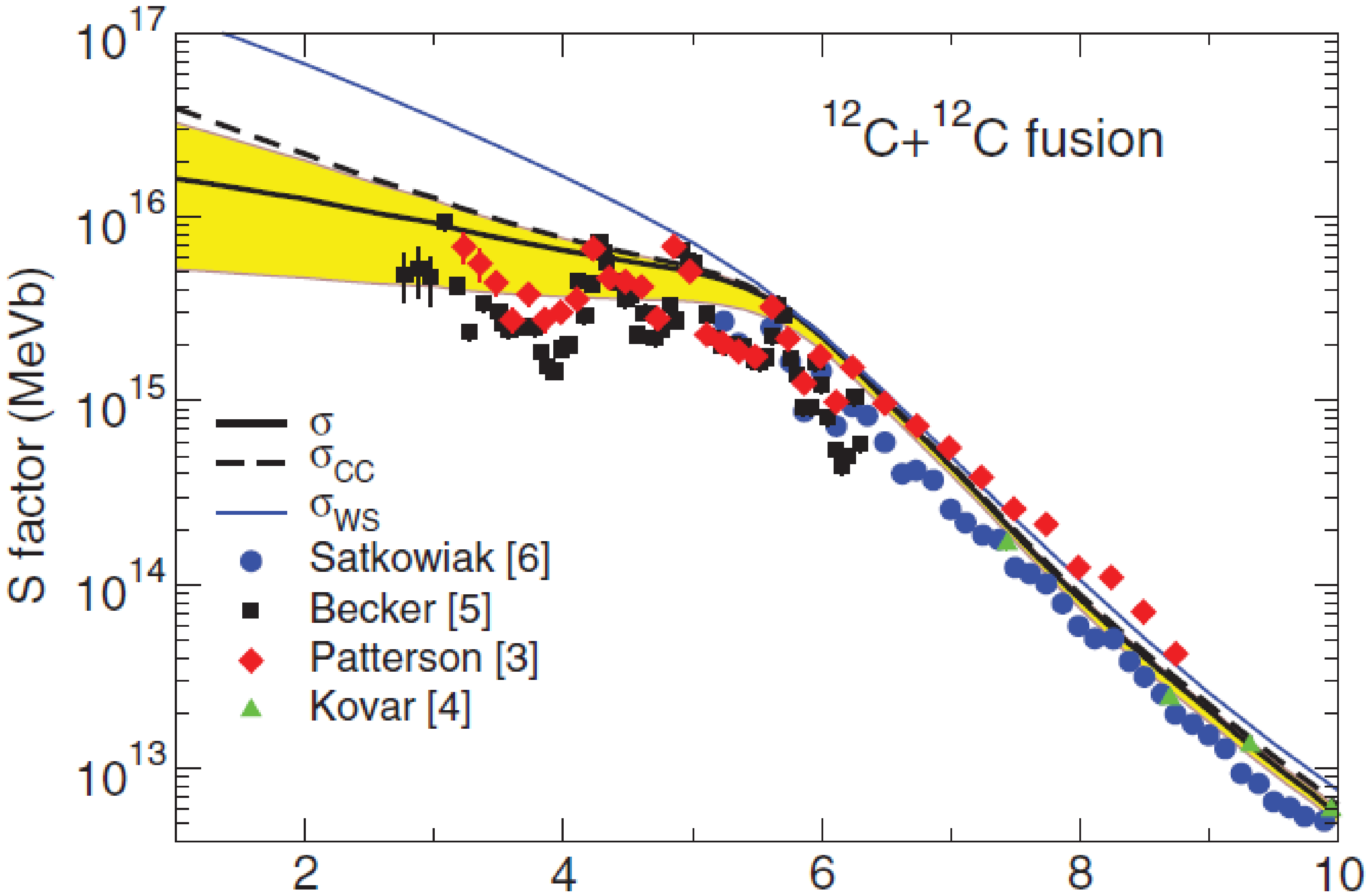


16O + 154Sm
no coupl.
with coupl.
Wong's formula
σ_{fus} (mb)
$E_{c.m.}$ (MeV)
10^3
10^2
10^1
10^0
10^{-1}
10^{-2}
10^{-3}
50
55
60
65
70

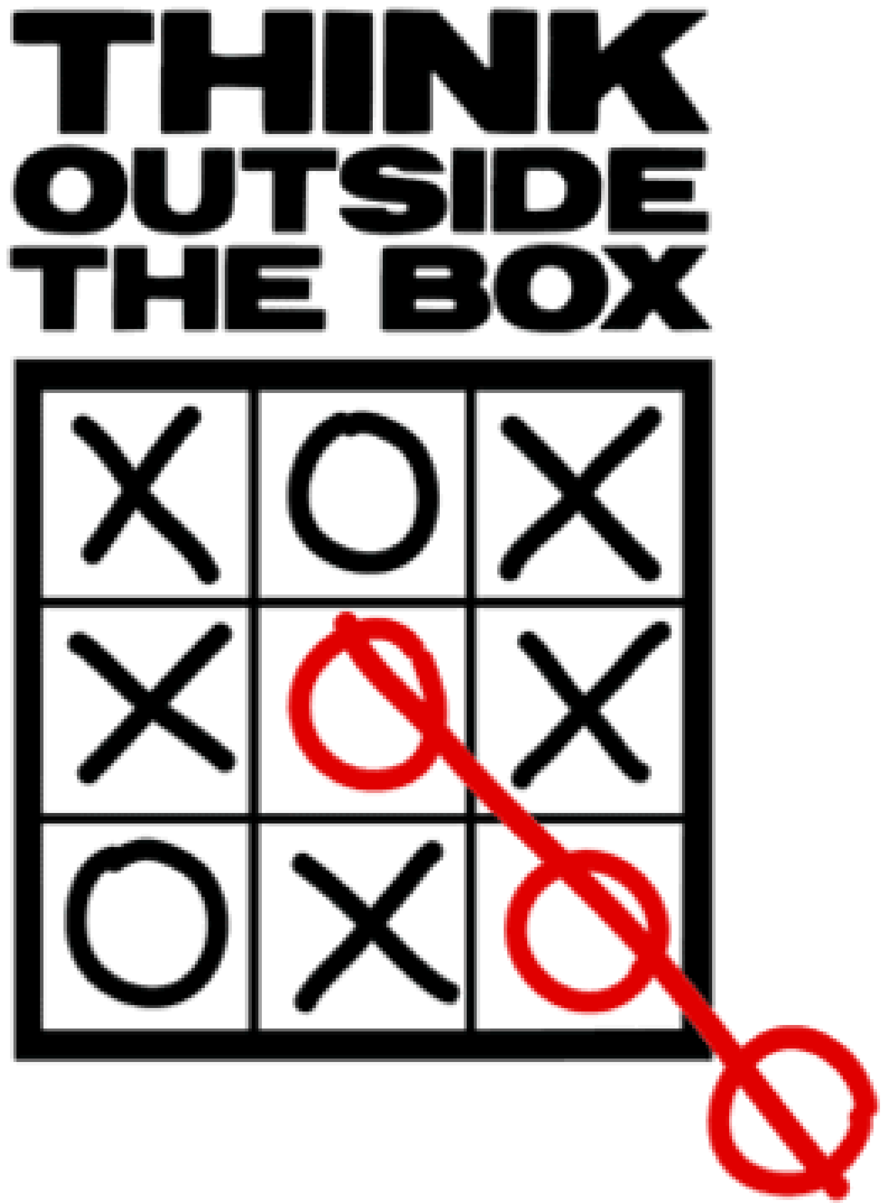
EXTRA SLIDES

Coupled-Channels Calculations for $^{12}C + ^{12}C$

Jiang, Esbensen et al., PRL **110** (2013) 072701

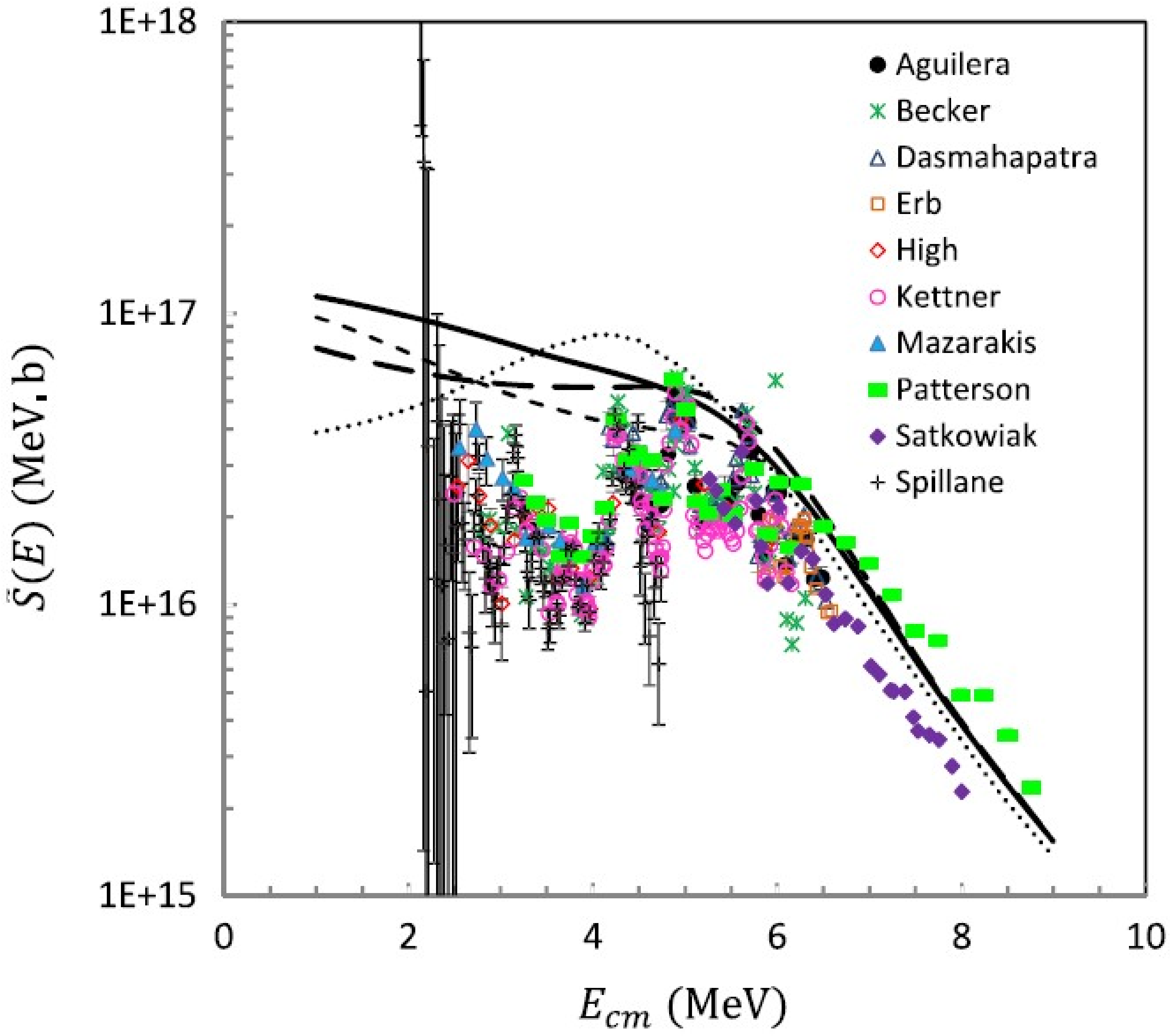


Ec.m. (MeV)

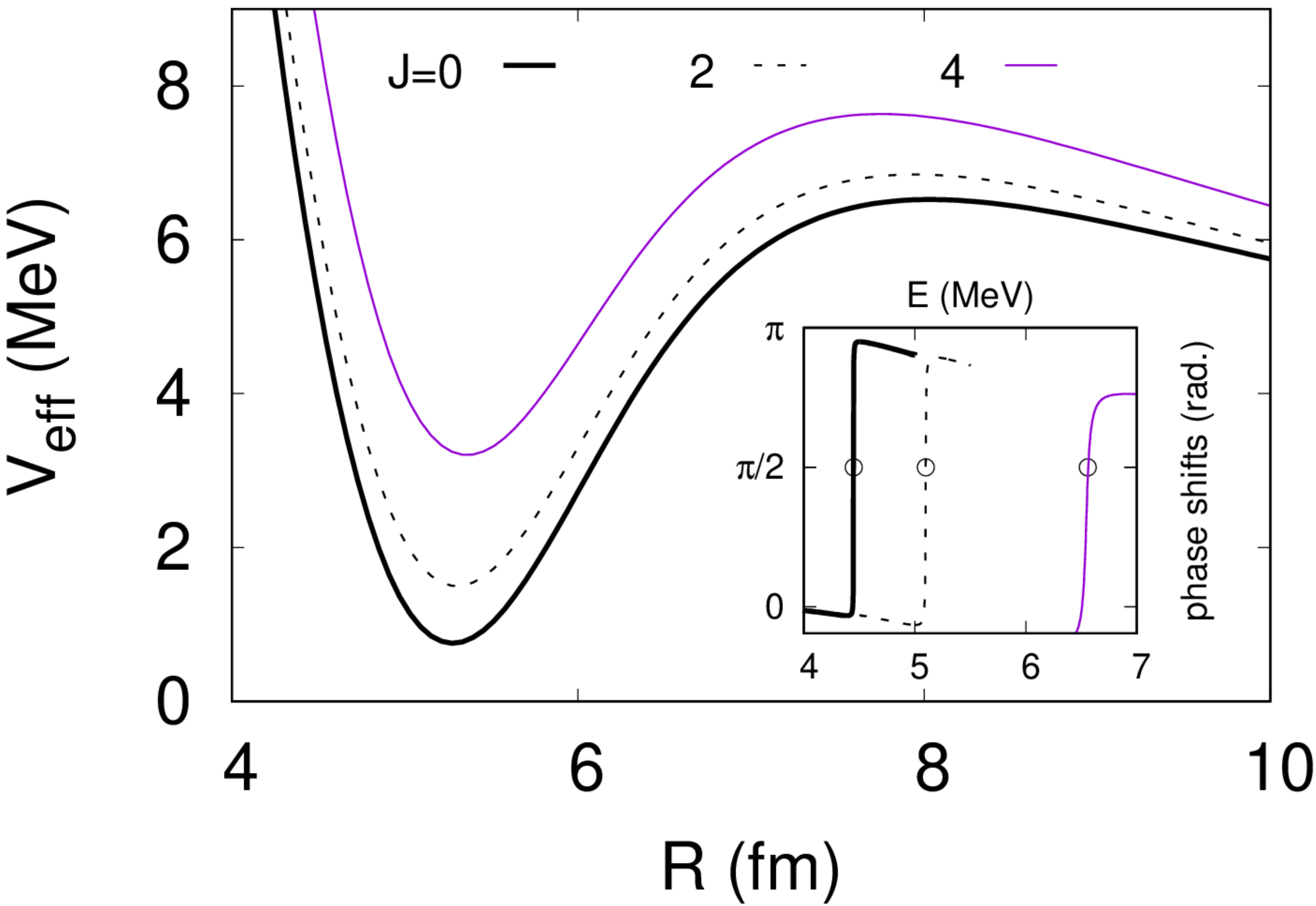

THINK
OUTSIDE
THE BOX

Coupled-Channels Calculations for $^{12}C + ^{12}C$

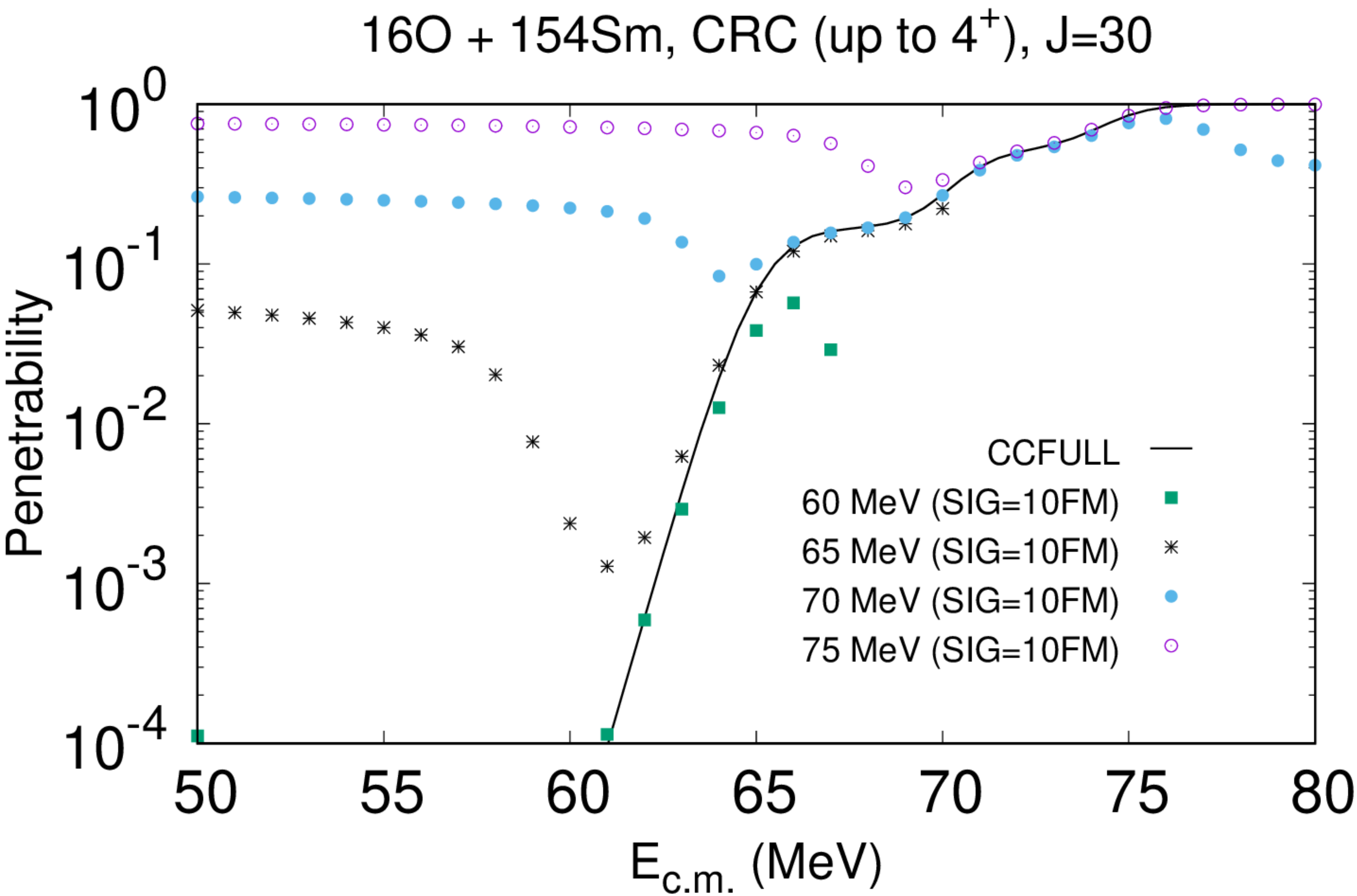
Assuncao & Descouvemont, PLB **723** (2013) 355



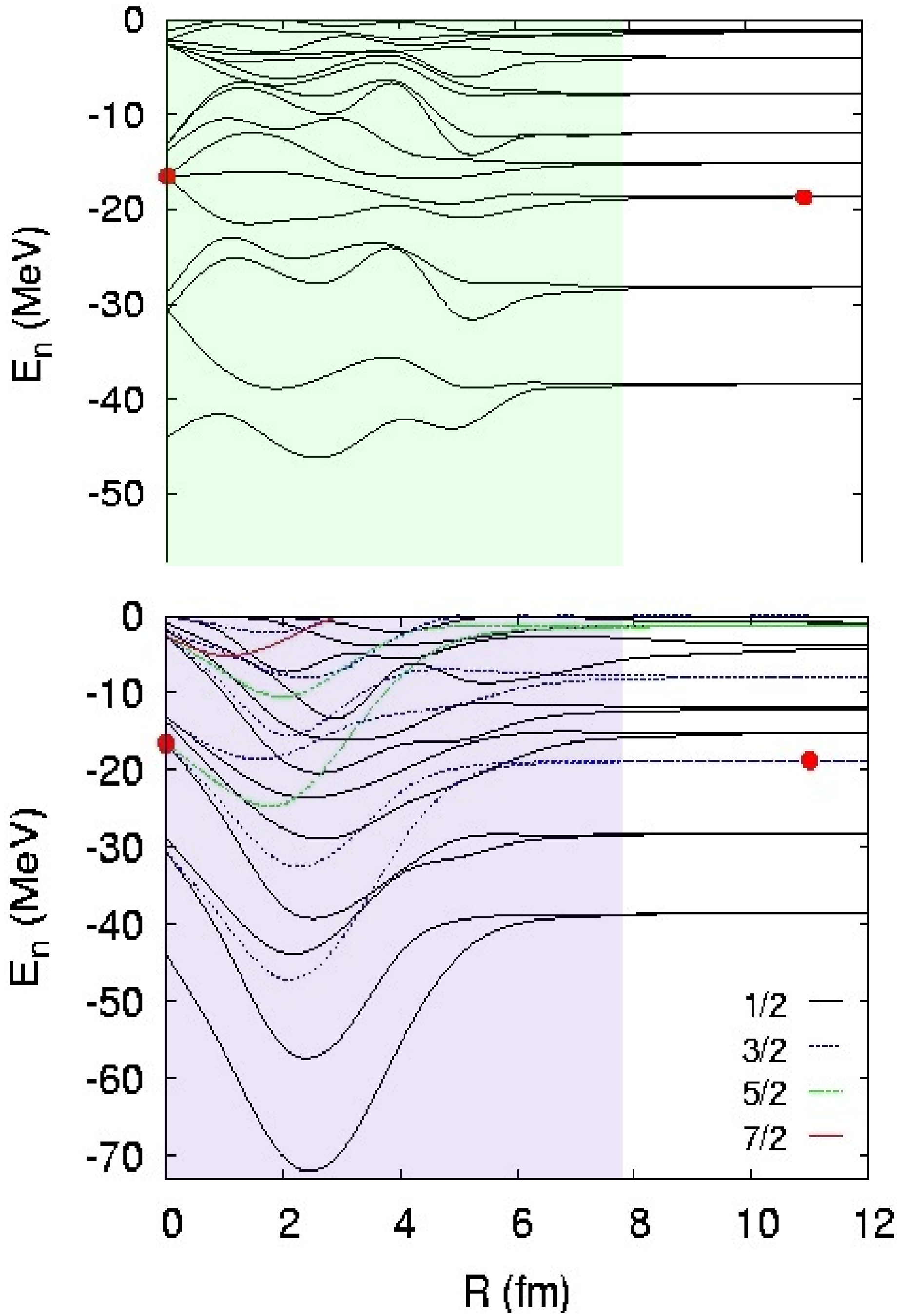
Phase shift analysis of effective potentials for $^{12}C + ^{12}C$

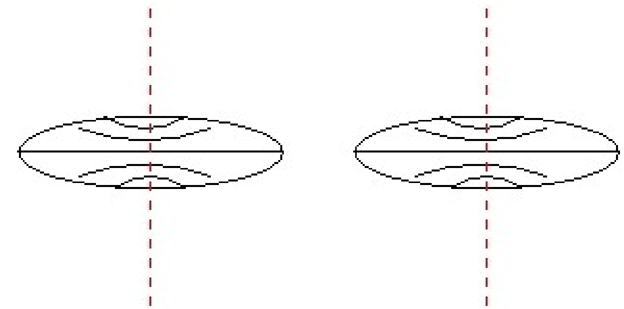


Wave-packet dynamics & stationary CRC



Sensitivity of Molecular Shell Structure to the ^{12}C Alignment

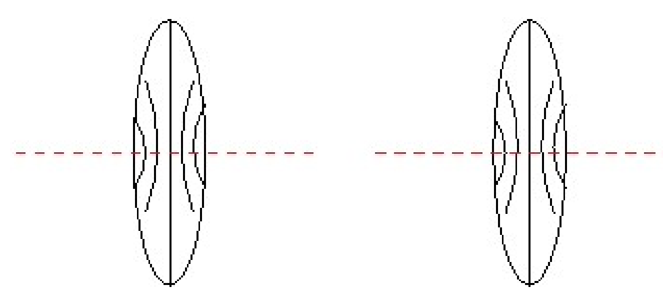




$$V = \sum_{s=1}^{2} e^{-i\mathbf{R}_s\hat{k}}\,\hat{U}(\Omega_s)\,V_s\,\hat{U}^{-1}(\Omega_s)\,e^{i\mathbf{R}_s\hat{k}}$$

$$V_s \approx \sum_{\nu\mu}^{N} |s\nu\rangle\, V^s_{\nu\mu}\, \langle s\mu|$$

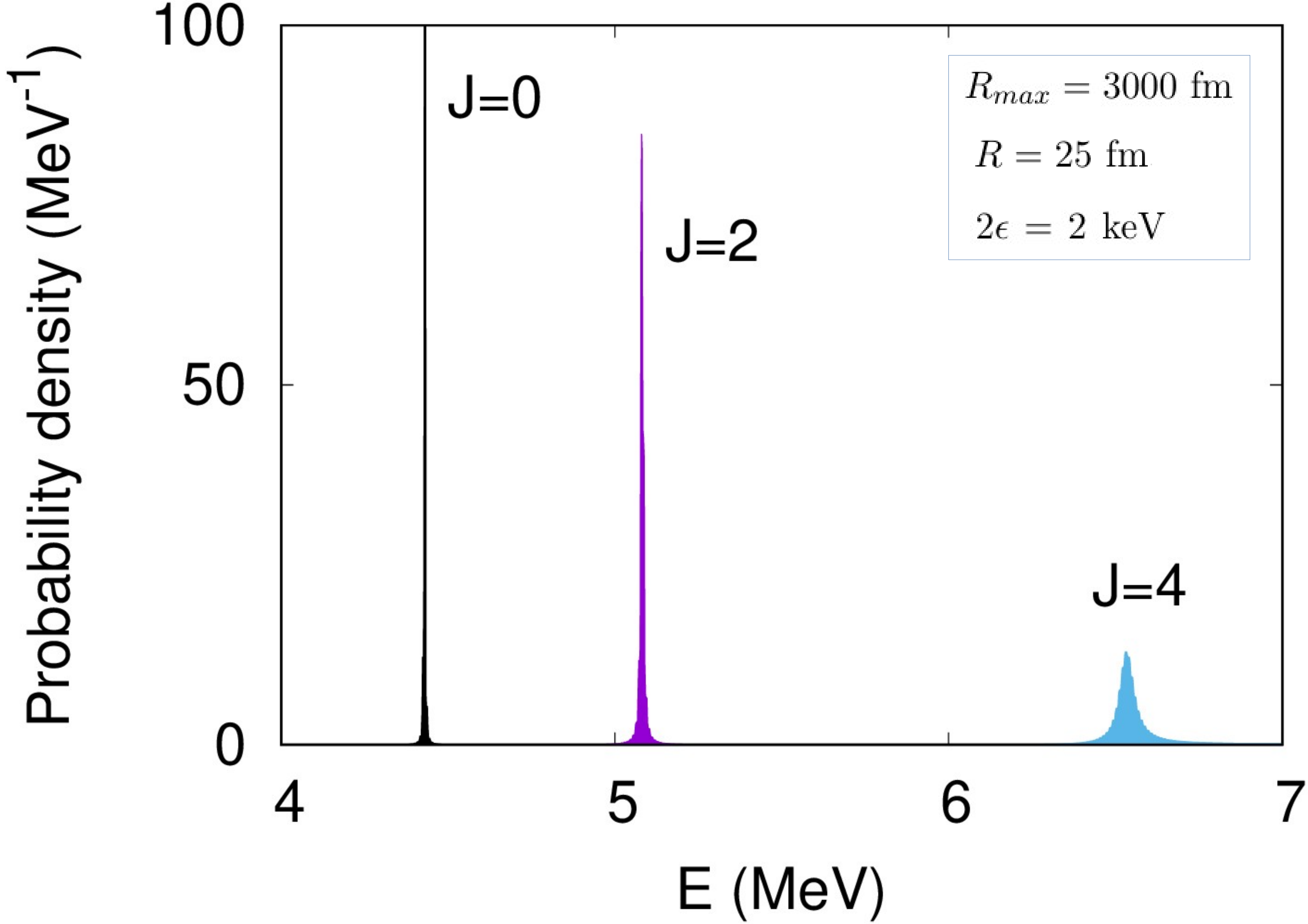
Potential Separable Expansion Method



AD-T, PRL **101** (2008) 122501

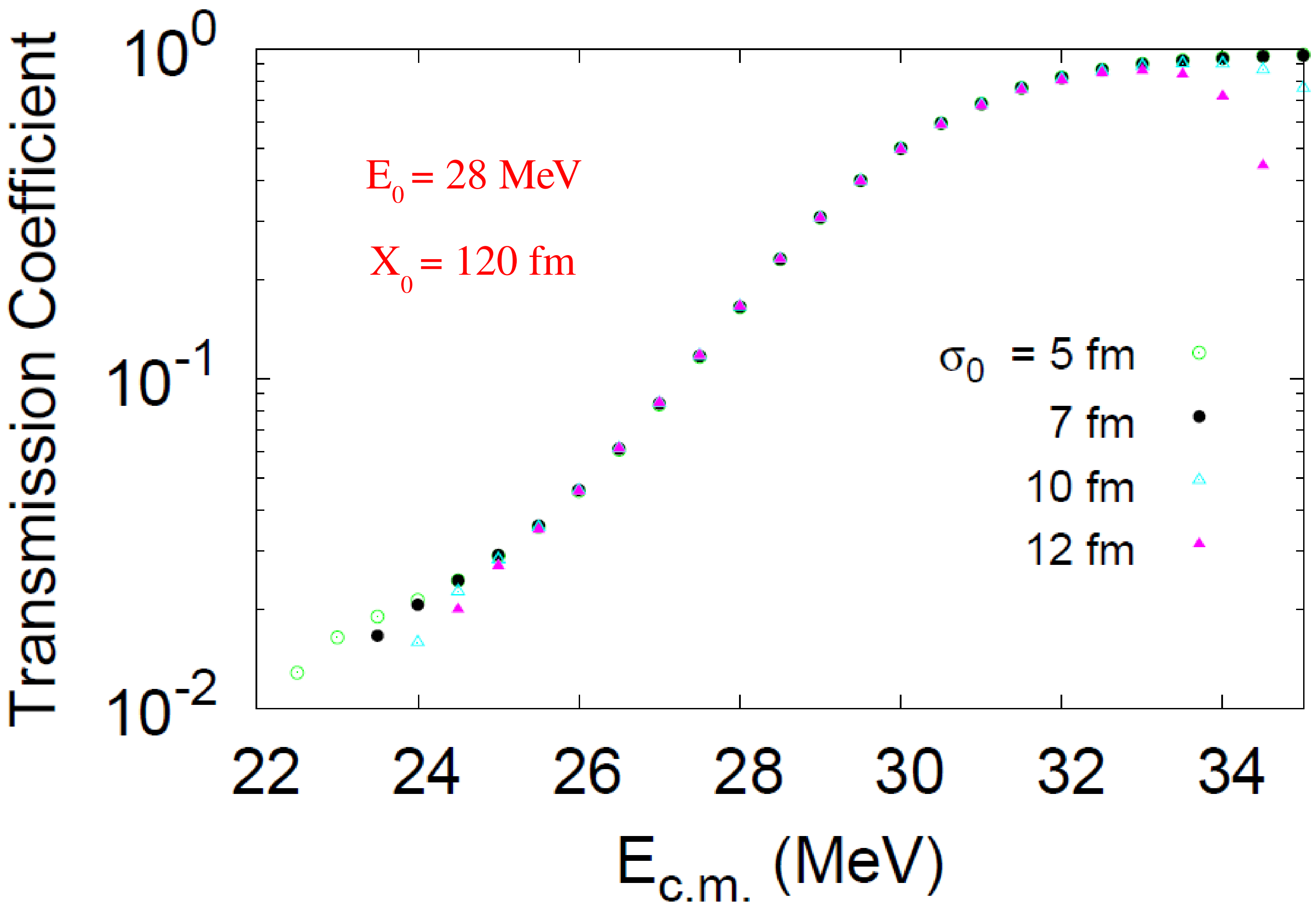
Probability Density Function

AD-T & Tostevin, arXiv: 1809.10517v1



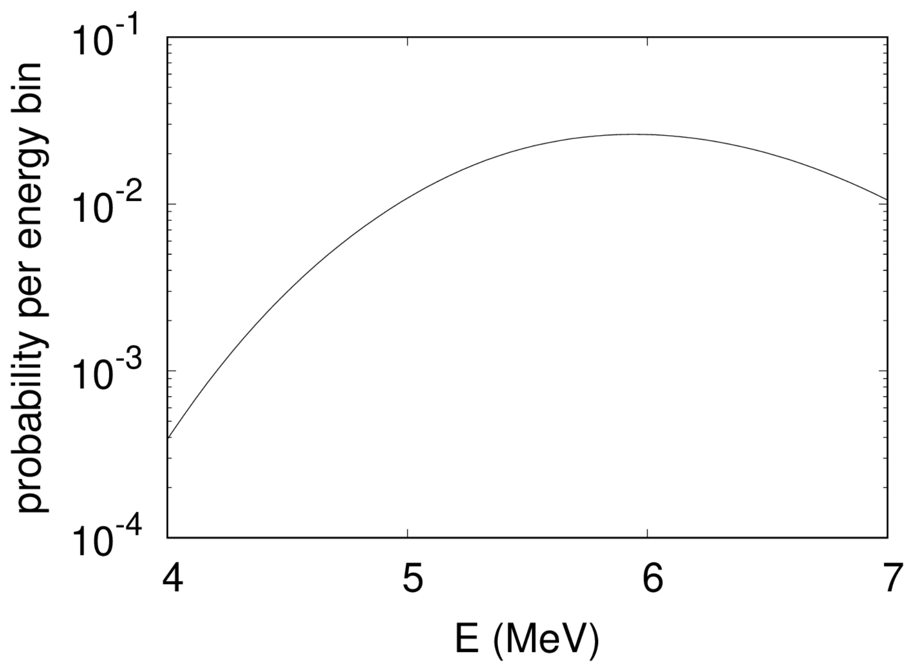
Results

♦ Energy-resolved **total transmission** for different values of the **spatial width** of the **initial wave packet**



Energy Projection of the Wave Function

Schafer & Kulander,
PRA **42** (1990) 5794



- **Energy spectra** of $\Psi(t)$ as expectation values of **the window operator**

$$\hat{\Delta}(E_k, n, \epsilon) \equiv \frac{\epsilon^{2^n}}{(\hat{\mathcal{H}} - E_k)^{2^n} + \epsilon^{2^n}}$$

$$E_{k+1} = E_k + 2\epsilon$$

- $\mathcal{P}(E_k) = \langle \Psi | \hat{\Delta} | \Psi \rangle$, for instance, **n = 2** :

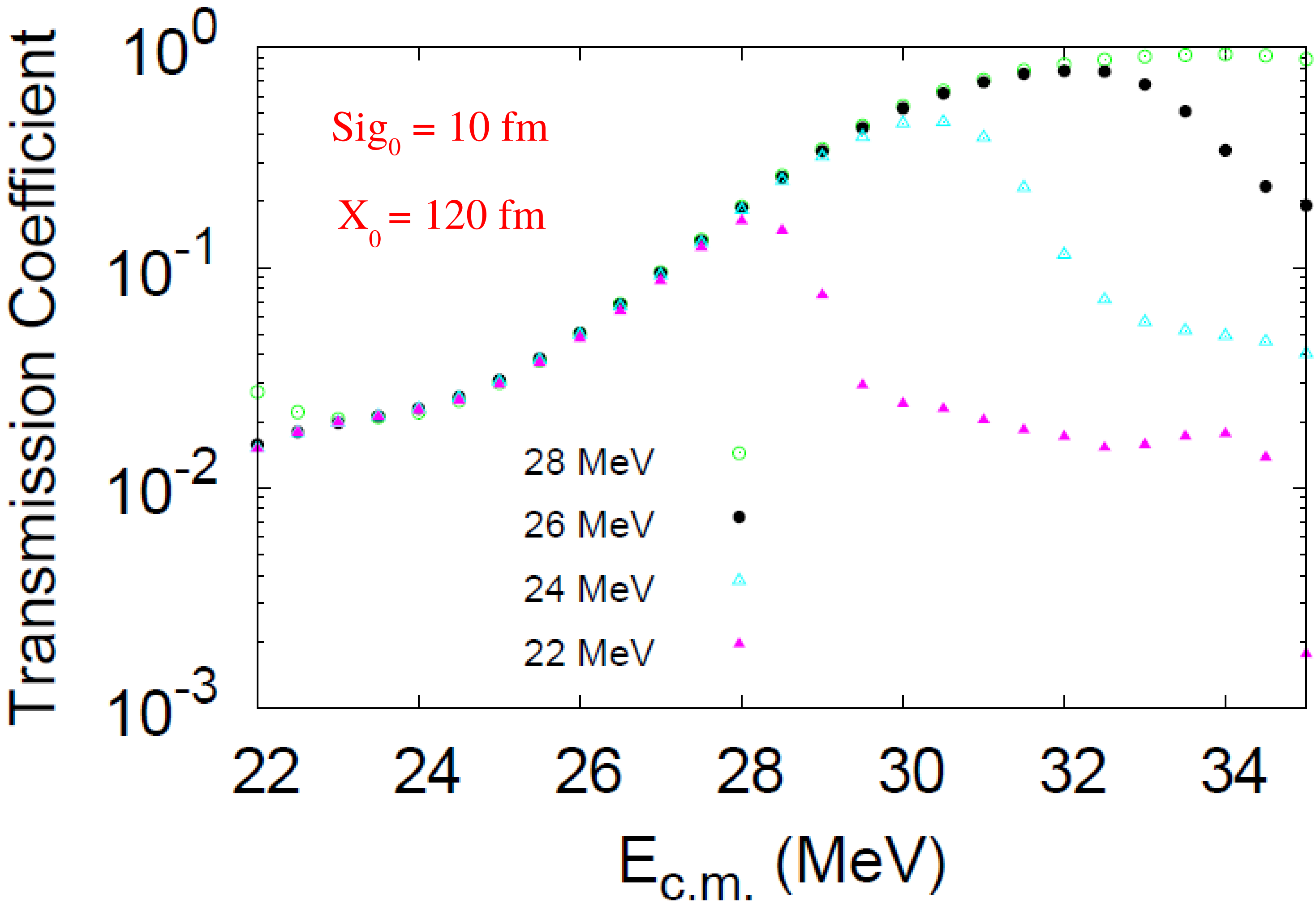
$$(\hat{H} - E_k + \sqrt{i}\,\epsilon)(\hat{H} - E_k - \sqrt{i}\,\epsilon)\,|\chi_k\rangle = \epsilon^2\,|\Psi\rangle$$

$\rightarrow$

$$\mathcal{P}(E_k) = \langle \chi_k | \chi_k \rangle$$

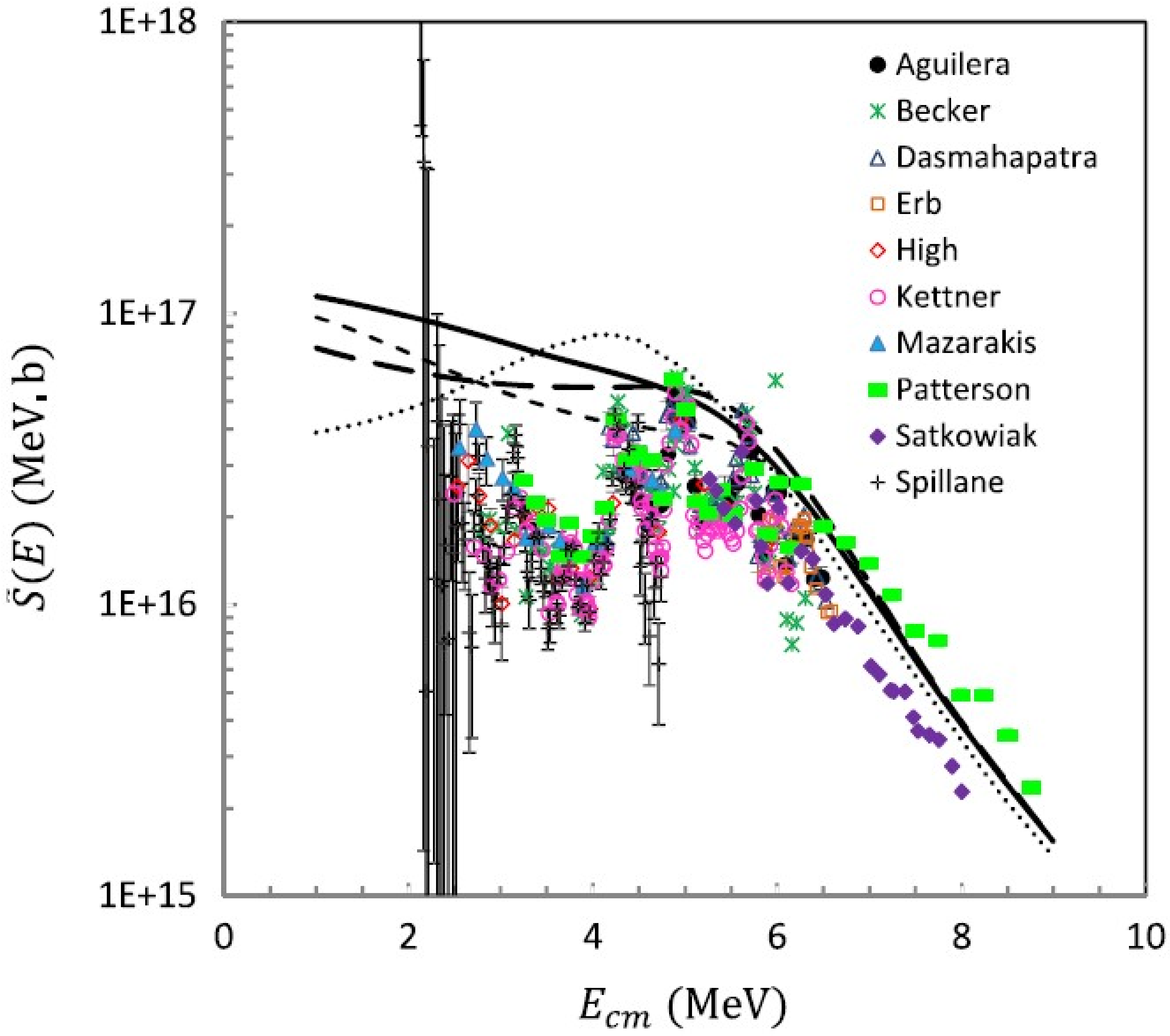
Results

♦ Energy-resolved **total transmission** for different values of the **mean energy** of the **initial wave packet**



Coupled-Channels Calculations for $^{12}C + ^{12}C$

Assuncao & Descouvemont, PLB **723** (2013) 355



Fusion Cross Section & Astrophysical S-Factor

$$S(E) = \sigma(E)\, E\, \exp(2\pi\eta)$$

Structural factor [MeV barn]

Fusion cross section [barn = 10^{-28} m^2]

$$\eta = \left(\frac{\mu}{2}\right)^{1/2} \frac{Z_1 Z_2 e^2}{\hbar E^{1/2}}$$

Sommerfeld parameter

S(E) represents the fusion cross section free of Coulomb suppression, which is adequate for extrapolation towards stellar energies

Kinetic-Energy of Two Deformed Colliding Nuclei

Gatti *et al.*, JCP **123** (2005) 174311

$$\frac{2\hat{T}}{\hbar^2} = -\frac{1}{\mu}\frac{\partial^2}{\partial R^2} + \left(\frac{1}{I_1} + \frac{1}{\mu R^2}\right)\hat{j}_1^2 + \left(\frac{1}{I_2} + \frac{1}{\mu R^2}\right)\hat{j}_2^2$$
$$+ \frac{1}{\mu R^2}\left[\hat{j}_{1,+}\hat{j}_{2,-} + \hat{j}_{1,-}\hat{j}_{2,+} + J(J+1)\right.$$
$$\left. - 2k_1^2 - 2k_1 k_2 - 2k_2^2\right] - \frac{C_+(J,K)}{\mu R^2}(\hat{j}_{1,+} + \hat{j}_{2,+})$$
$$- \frac{C_-(J,K)}{\mu R^2}(\hat{j}_{1,-} + \hat{j}_{2,-})$$

Coriolis interaction

μ is the reduced mass for the radial motion,
I_i is the ^{12}C rotational inertia,
J is the total angular momentum with projection $K = k_1 + k_2$,
$C_\pm(J,K) = \sqrt{J(J+1) - K(K\pm 1)}$,
$\hat{j}_i^2 = -\frac{1}{\sin\theta_i}\frac{\partial}{\partial\theta_i}\sin\theta_i\frac{\partial}{\partial\theta_i} + \frac{k_i^2}{\sin^2\theta_i}$,
$\hat{j}_{i,\pm} = \pm\frac{\partial}{\partial\theta_i} - k_i\cot\theta_i$, with $k_i \rightarrow k_i \pm 1$.

Initial state $\Psi(t=0)$: the [12]C nuclei are well separated

$$\Psi_0(R,\theta_1,k_1,\theta_2,k_2) = \chi_0(R)\,\psi_0(\theta_1,k_1,\theta_2,k_2),$$

Radial motion

Internal rotational motion

$$\chi_0(R) = (\sqrt{\pi}\sigma)^{-1/2}\,\exp\left[-\frac{(R-R_0)^2}{2\sigma^2}\right]e^{iP_0(R-R_0)},$$

$$\begin{aligned}\psi_0(\theta_1,k_1,\theta_2,k_2) &= \left[\zeta_{j_1,m_1}(\theta_1,k_1)\zeta_{j_2,m_2}(\theta_2,k_2)\right.\\ &\left.+(-1)^J\zeta_{j_2,-m_2}(\theta_1,k_1)\zeta_{j_1,-m_1}(\theta_2,k_2)\right]\\ &/\sqrt{2+2\,\delta_{j_1,j_2}\delta_{m_1,-m_2}},\end{aligned}$$

where $\zeta_{j,m}(\theta,k) = \sqrt{\frac{(2j+1)(j-m)!}{2\,(j+m)!}}\,P_j^m(\cos\theta)\,\delta_{km}$,
and P_j^m are associated Legendre functions.

Time Propagation of the Wave Function

$$|\Psi_J(t)\rangle = \underbrace{e^{-i\,\hat{H}\,t/\hbar}}_{\text{evolution operator}}\,|\Psi_J(0)\rangle$$

evolution operator

The evolution operator is represented as a convergent series of modified Chebyshev polynomials

Tannor, Quantum Mechanics from a Time-Dependent Perspective, USB, 2007

Power Spectrum of the Wave Function

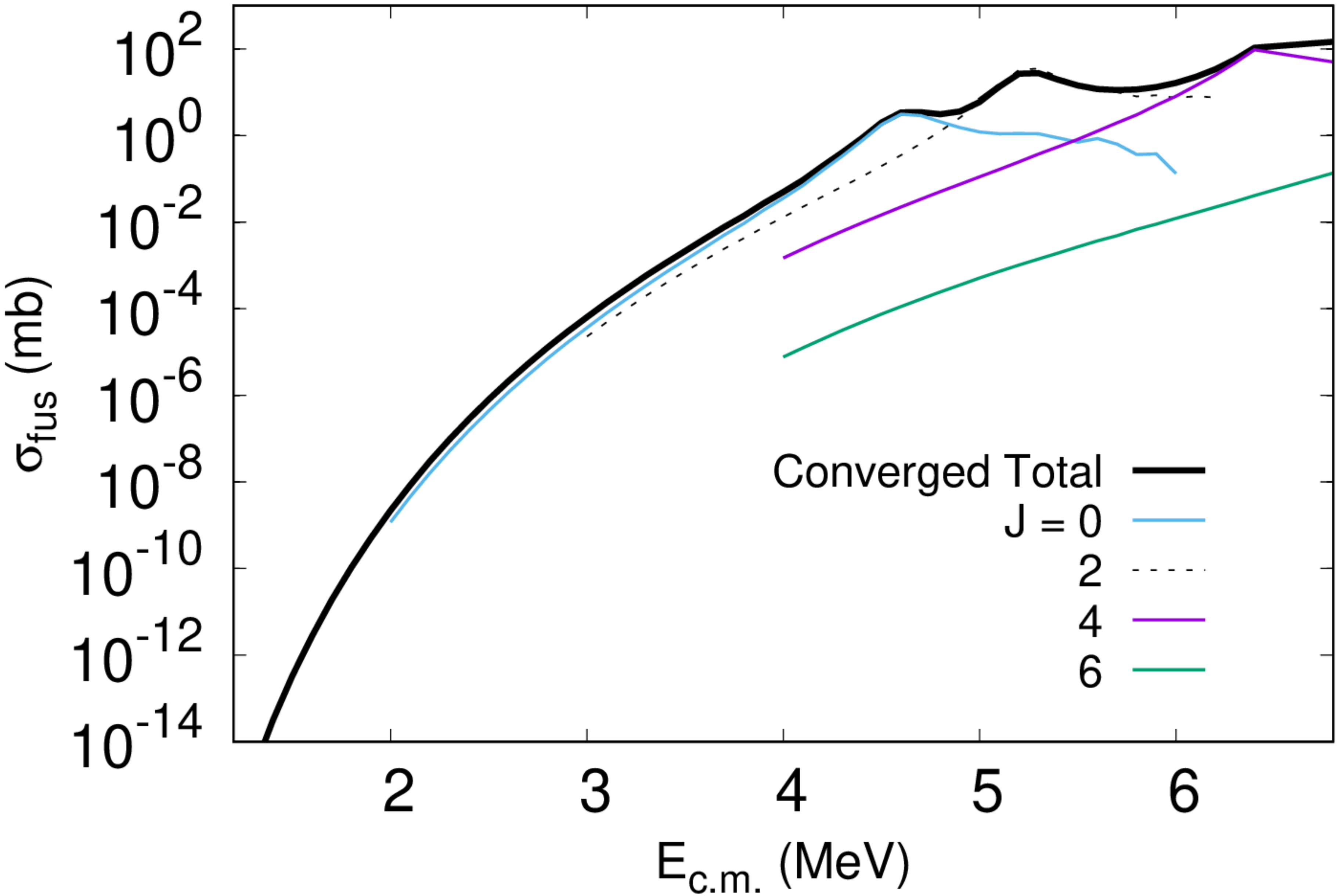
$$\mathcal{P}(E) = \langle \Psi(t) | \underbrace{\delta(E - \hat{H})}_{\text{Energy projector}} | \Psi(t) \rangle$$

Reflection & Transmission Coefficients

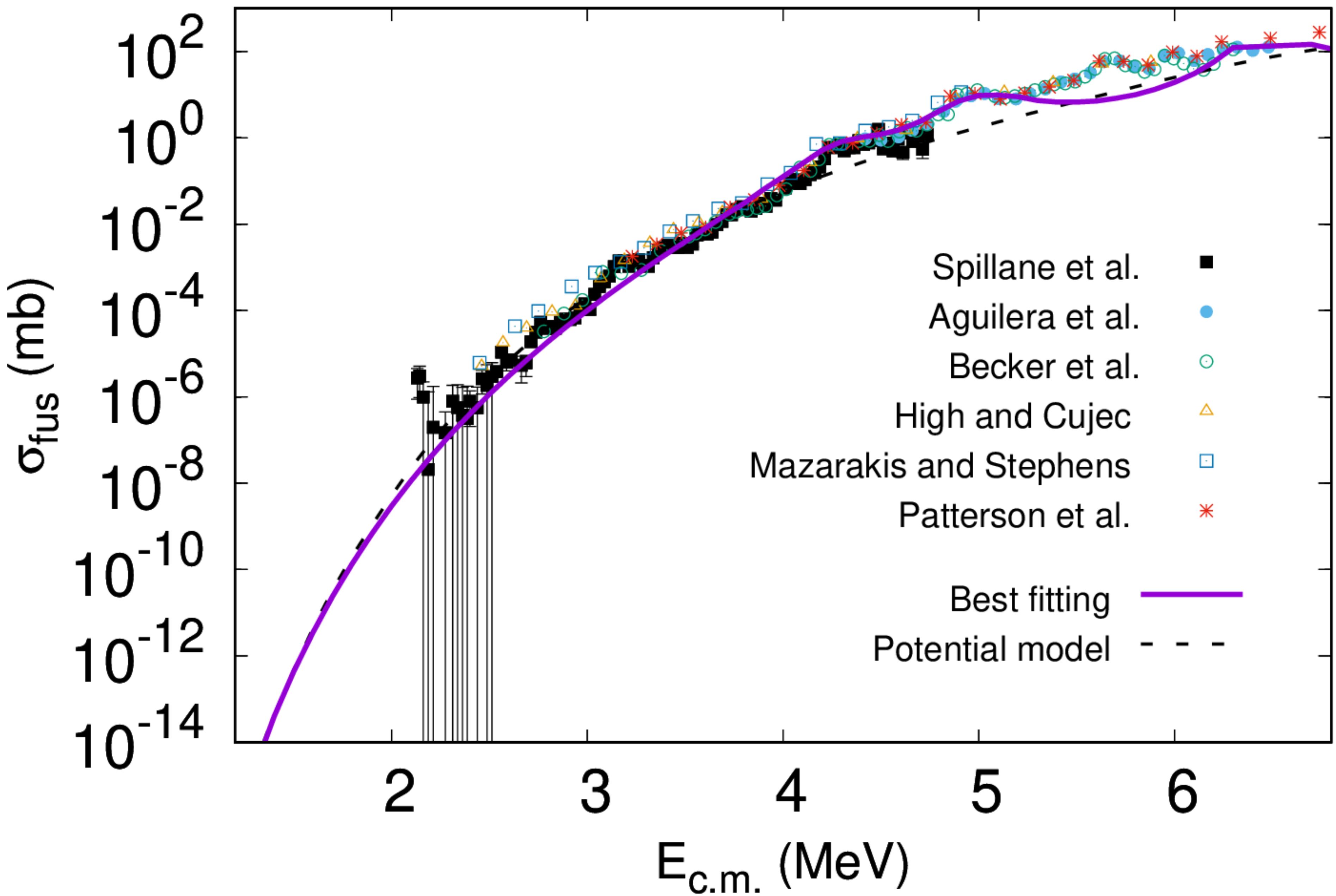
$$\mathcal{R}(E) = \frac{\mathcal{P}^{final}(E)}{\mathcal{P}^{initial}(E)}$$

$$T(E) = 1 - \mathcal{R}(E)$$

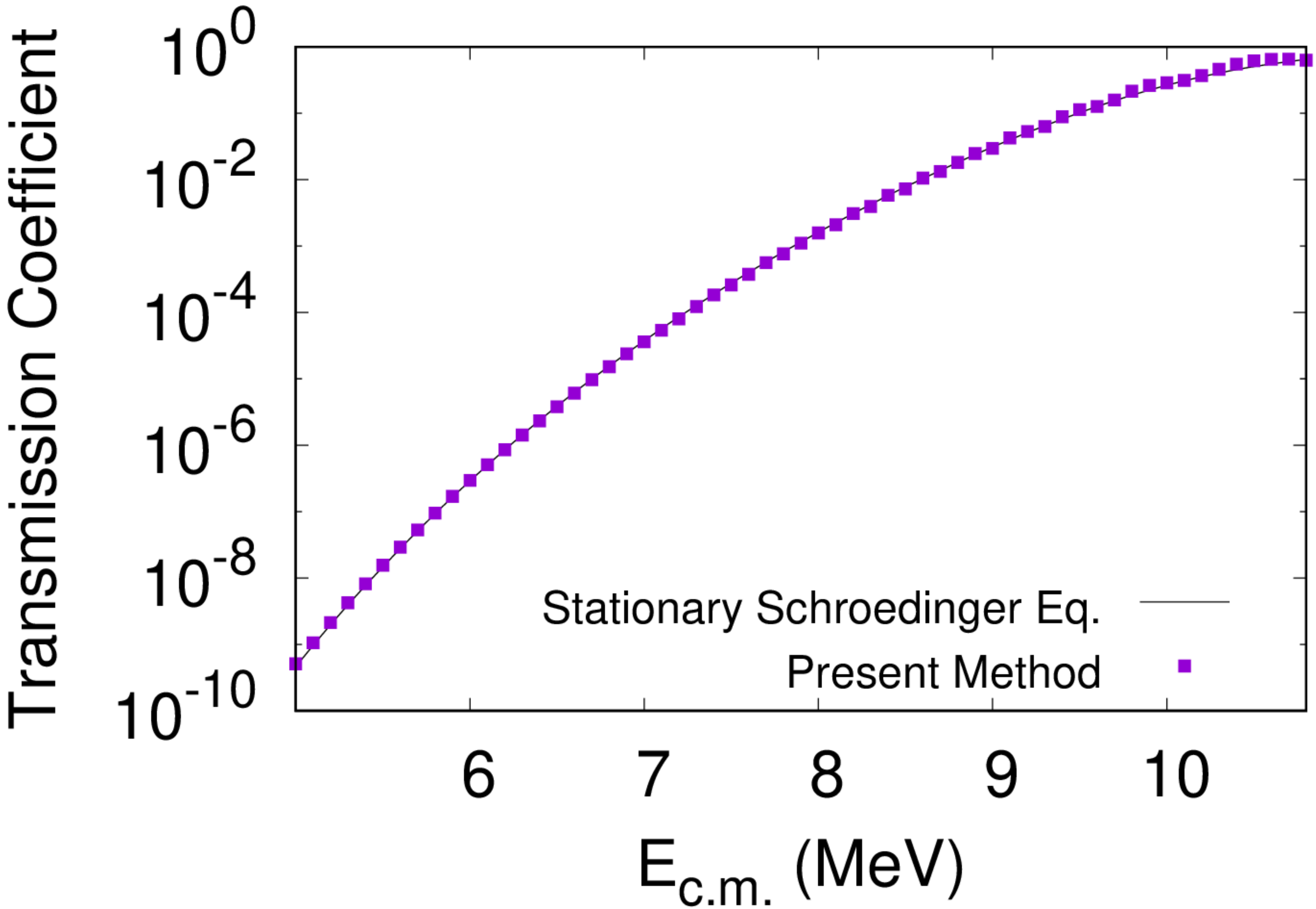
Fusion Excitation Function for $^{12}C + ^{12}C$



Fusion Excitation Function for $^{12}C + ^{12}C$



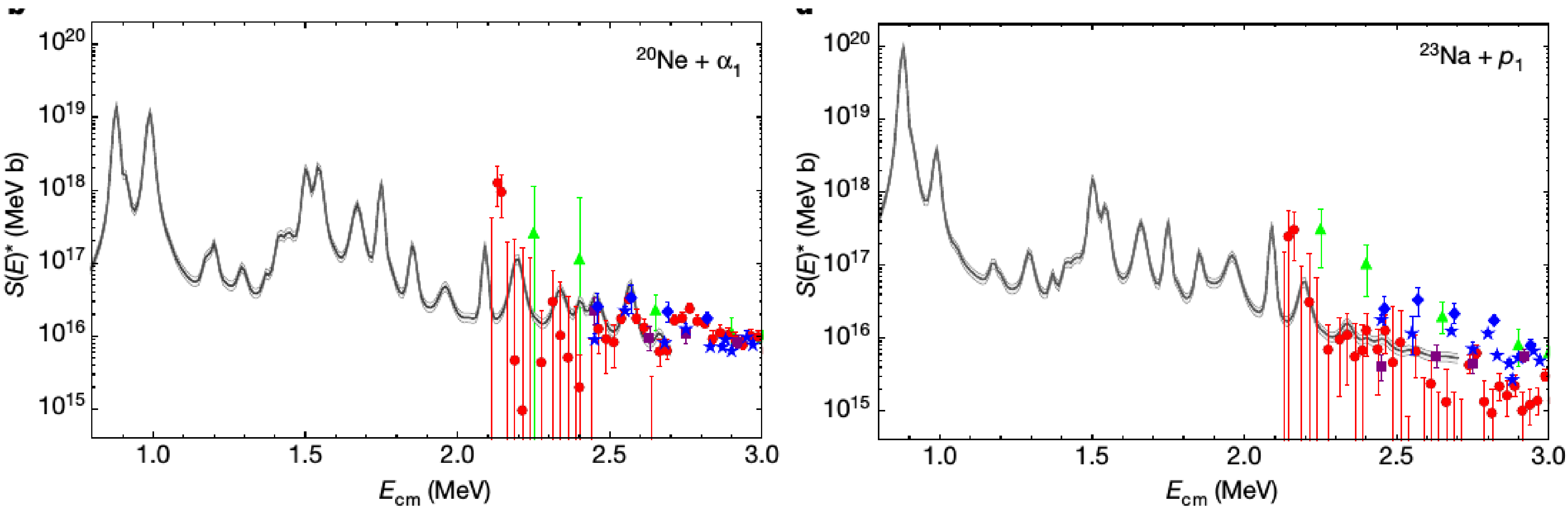
Transmission coefficients for $^{16}O + ^{16}O$ central collisions

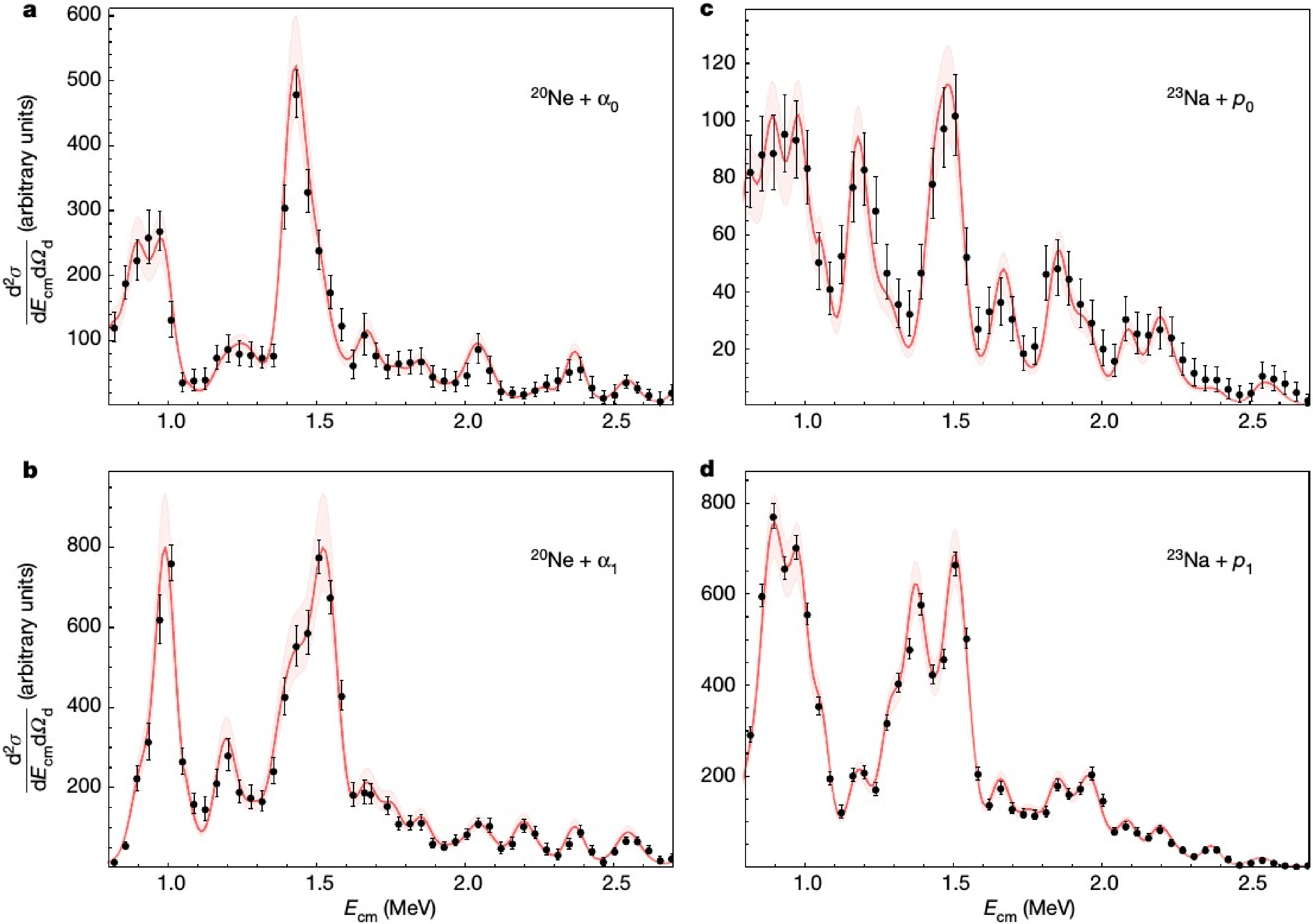


https://doi.org/10.1038/s41586-018-0149-4

An increase in the $^{12}C + ^{12}C$ fusion rate from resonances at astrophysical energies

A. Tumino[1,2]*, C. Spitaleri[2,3], M. La Cognata[2], S. Cherubini[2,3], G. L. Guardo[2,4], M. Gulino[1,2], S. Hayakawa[2,5], I. Indelicato[2], L. Lamia[2,3], H. Petrascu[4], R. G. Pizzone[2], S. M. R. Puglia[2], G. G. Rapisarda[2], S. Romano[2,3], M. L. Sergi[2], R. Spartá[2] & L. Trache[4]





a
b
c
d
20Ne + α0
20Ne + α1
23Na + p0
23Na + p1
d2σ / dEcm dΩd (arbitrary units)
Ecm (MeV)
600
500
400
300
200
100
800
120
1.0
1.5
2.0
2.5