



Low lying collective phenomena in $N \sim 90$ nuclei

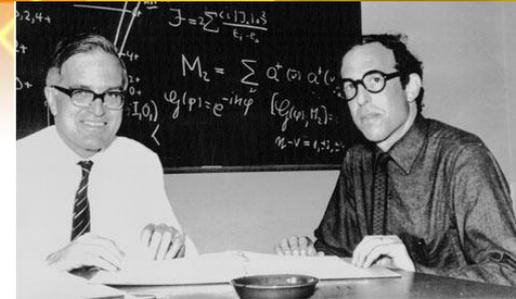
University of Zululand
Siyabonga Majola

UniZulu 2019

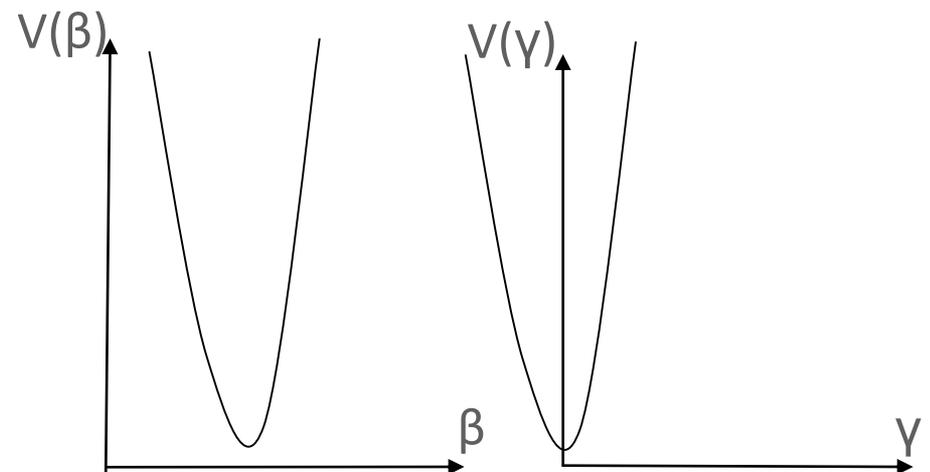
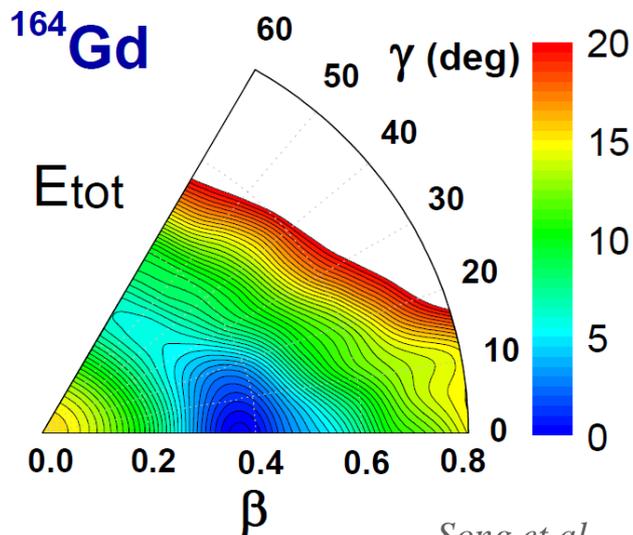
Background and motivation

- Bohr Approach

$$H = T_{vib} + T_{rot} + V(\beta, \gamma)$$



$$H = -\frac{\hbar^2}{2} \left[\frac{1}{B^2} \frac{\partial^2}{\partial \beta^2} + \frac{1}{B_\gamma} \frac{1}{\gamma} \frac{\partial}{\partial \gamma} \left(\gamma \frac{\partial}{\partial \gamma} \right) \right] + \frac{\hbar^2}{2\mathfrak{I}} (I(I+1) - K^2) + \frac{K^2 \hbar^2}{2\mathfrak{I}_z} + \frac{1}{2} C_\beta (\beta_2 - \beta_0)^2 + \frac{1}{2} C_\gamma \gamma^2$$



Background and motivation

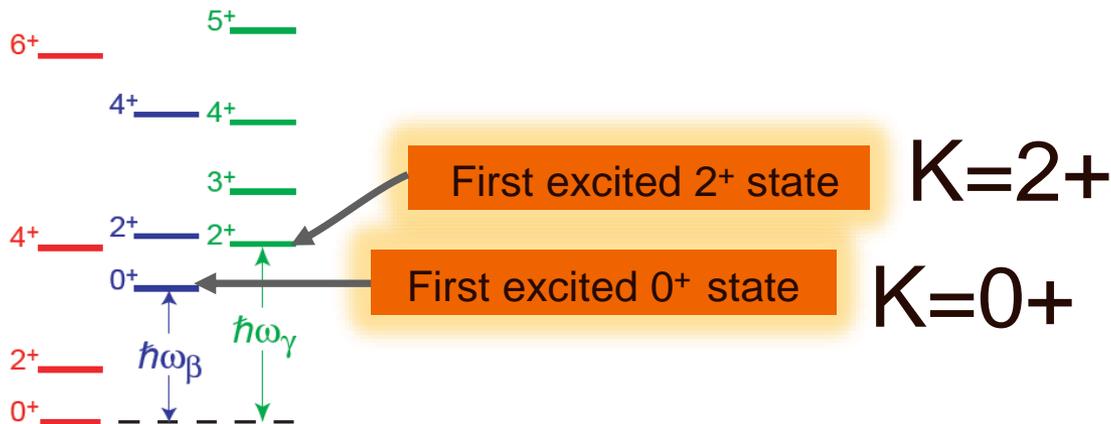
- Early success of the Bohr Approach

Early success of the Bohr
in the Rare earth Region

β vibration



γ vibration



Background and motivation

- 0^+ bands do not ALL have the characteristics predicted for β -bands

Characterization of the β vibration and 0_2^+ states in deformed nuclei

P E Garrett

Lawrence Livermore National Laboratory, Livermore, CA 94551, USA

Received 26 October 2000, in final form 7 November 2000

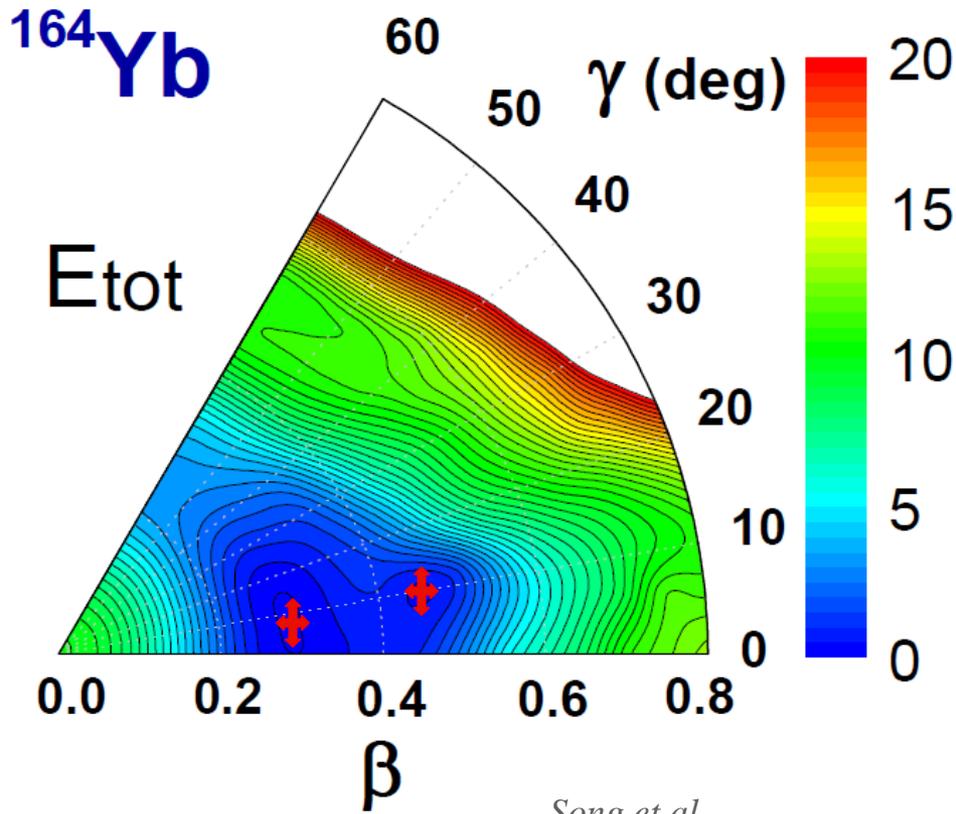
Abstract

A summary of the experimental properties of the first excited 0^+ states in deformed rare-earth nuclei is presented. By appealing to the original definition of a β vibration laid down in the Bohr–Mottelson picture, it is re-emphasized that most of the 0_2^+ states are not β vibrations. A consideration of all available data, especially that from transfer reactions, and of microscopic calculations of 0^+ states underscores the need to consider the role of pairing in the description, and labelling, of these states.

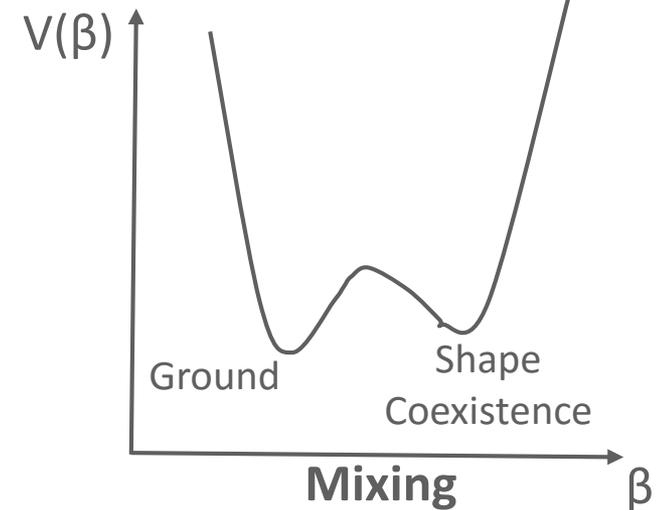
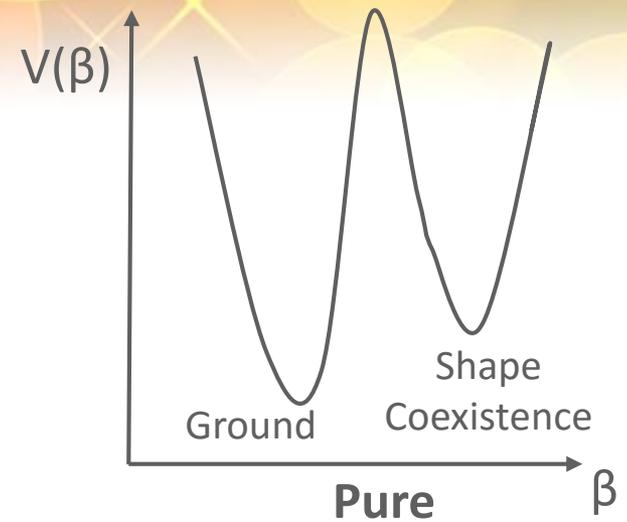


Background and motivation

- Shape Coexistence



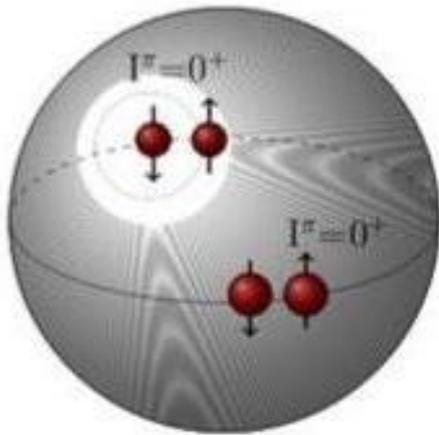
Song et al



Background and motivation

- Pairing

- Further confusion caused by pairing in the $N=90$, $A \approx 160$
 - quadrupole pairing
- Give rise to 0^+ ground states in even-even



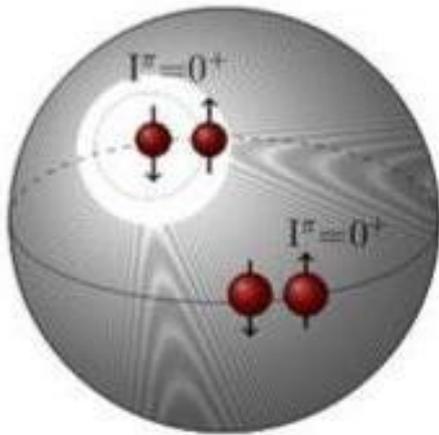
Monopole term

$$\Delta = G_0 \sum_{\nu'} U_{\nu'} V_{\nu'}$$

Background and motivation

- Pairing

- Further confusion caused by pairing in the N=90, A ≈ 160
- quadrupole pairing
- Give rise to 0⁺ ground states in even-even



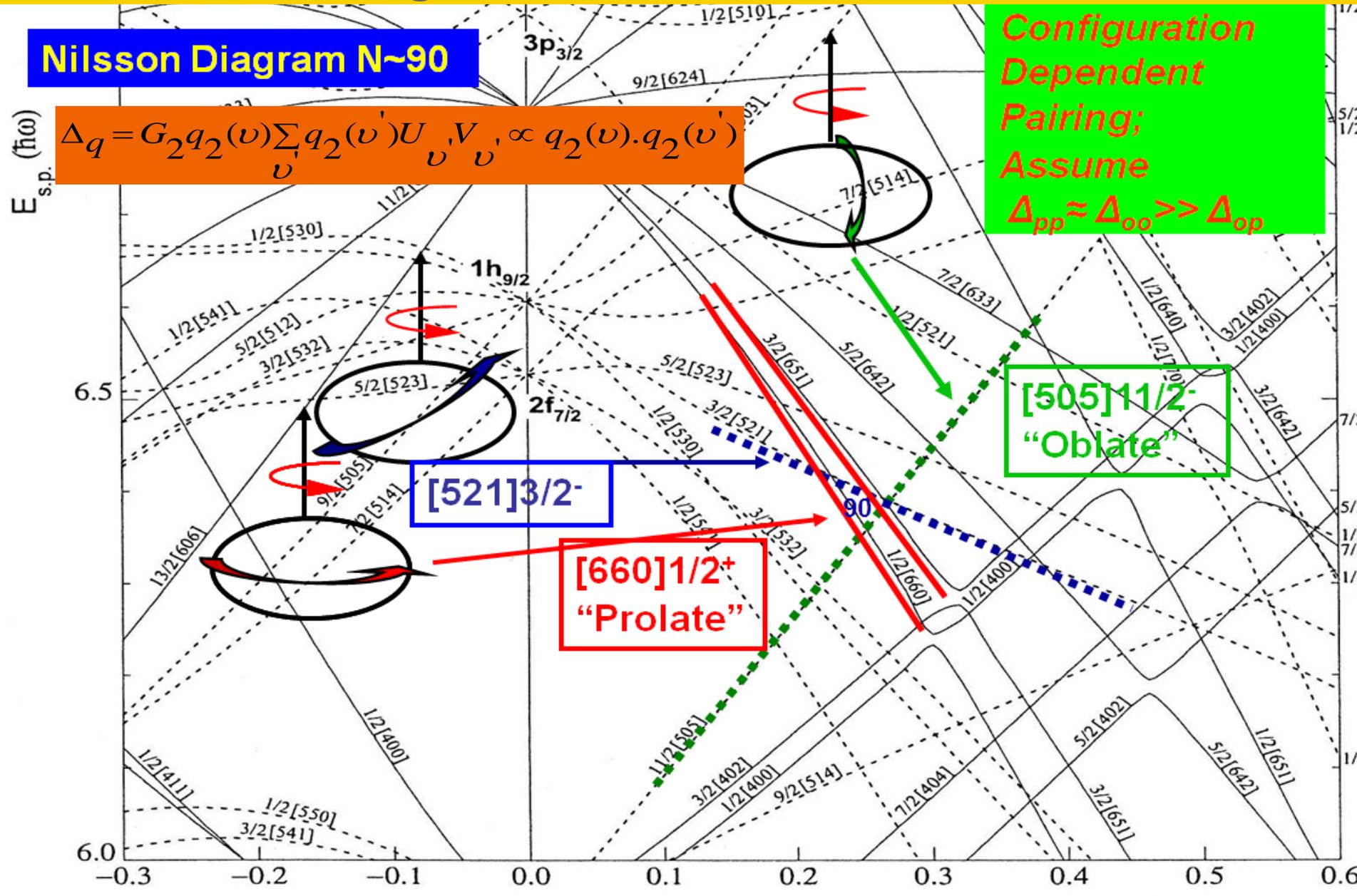
$$\Delta = \overbrace{G_0 \sum_{\nu'} U_{\nu'} V_{\nu'}}^{\text{Monopole term}} + \overbrace{G_2 q_2(\nu) \sum_{\nu'} q_2(\nu') U_{\nu'} V_{\nu'}}^{\text{Quadrupole term}}$$

Background and motivation

Nilsson Diagram N~90

$$\Delta_q = G_2 q_2(v) \sum_{v'} q_2(v') U_{v'v} \propto q_2(v) \cdot q_2(v')$$

Configuration
Dependent
Pairing;
Assume
 $\Delta_{pp} \approx \Delta_{oo} \gg \Delta_{op}$



[521]3/2-

[660]1/2+
"Prolate"

[505]11/2-
"Oblate"

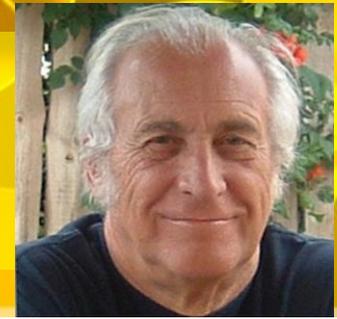
90

Background and motivation

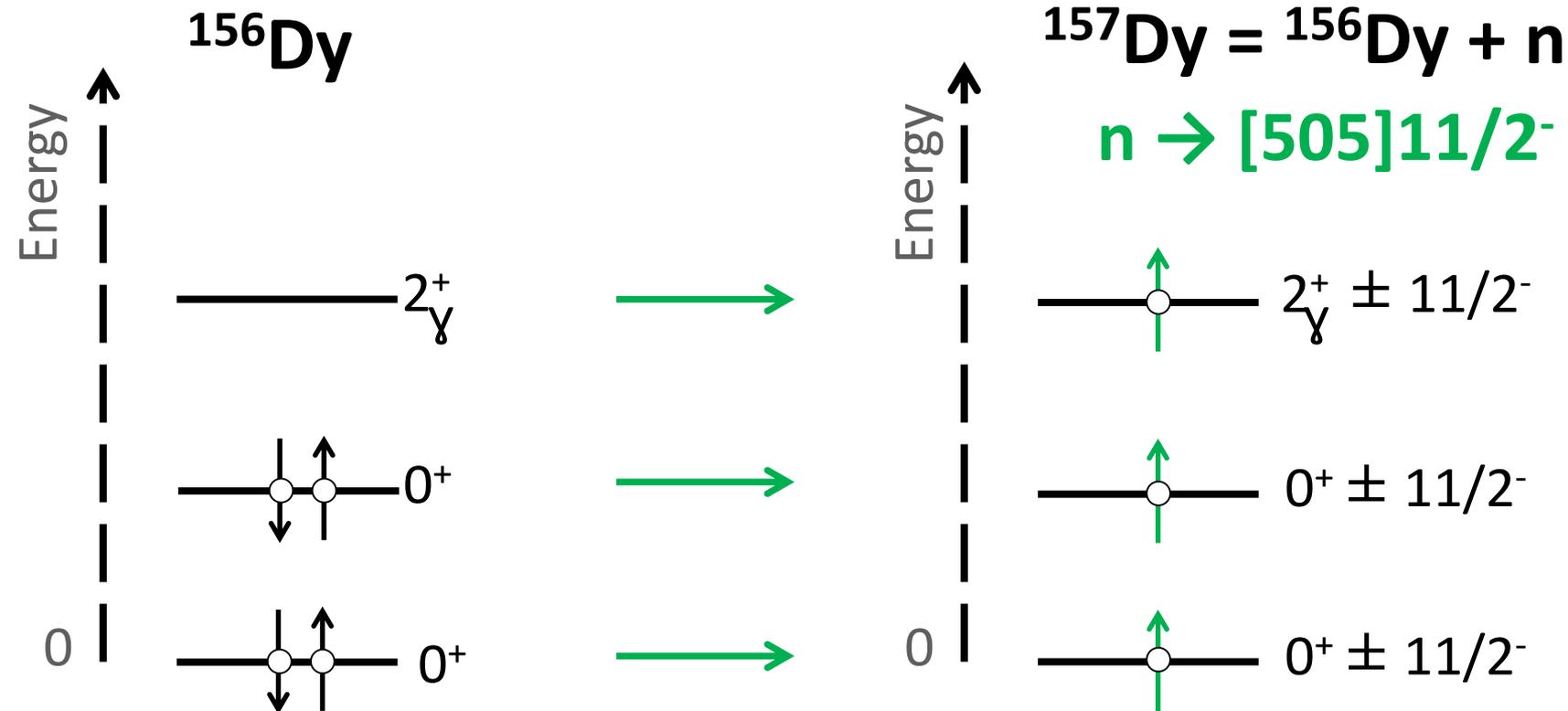
- Three modes of excitation, which could lead to a formation of a first-excited $K^\pi = 0^+$ bands.
- Are the first excited zero plus states β , **shape co-existence** or **Quadrupole pairing** bands (**Second vacuum bands**)??

Background and motivation

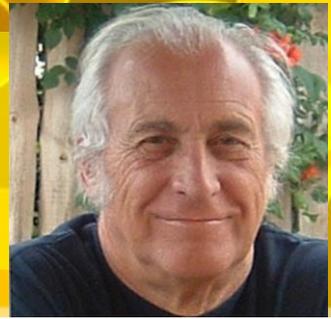
- Quadrupole pairing Hypothesis test



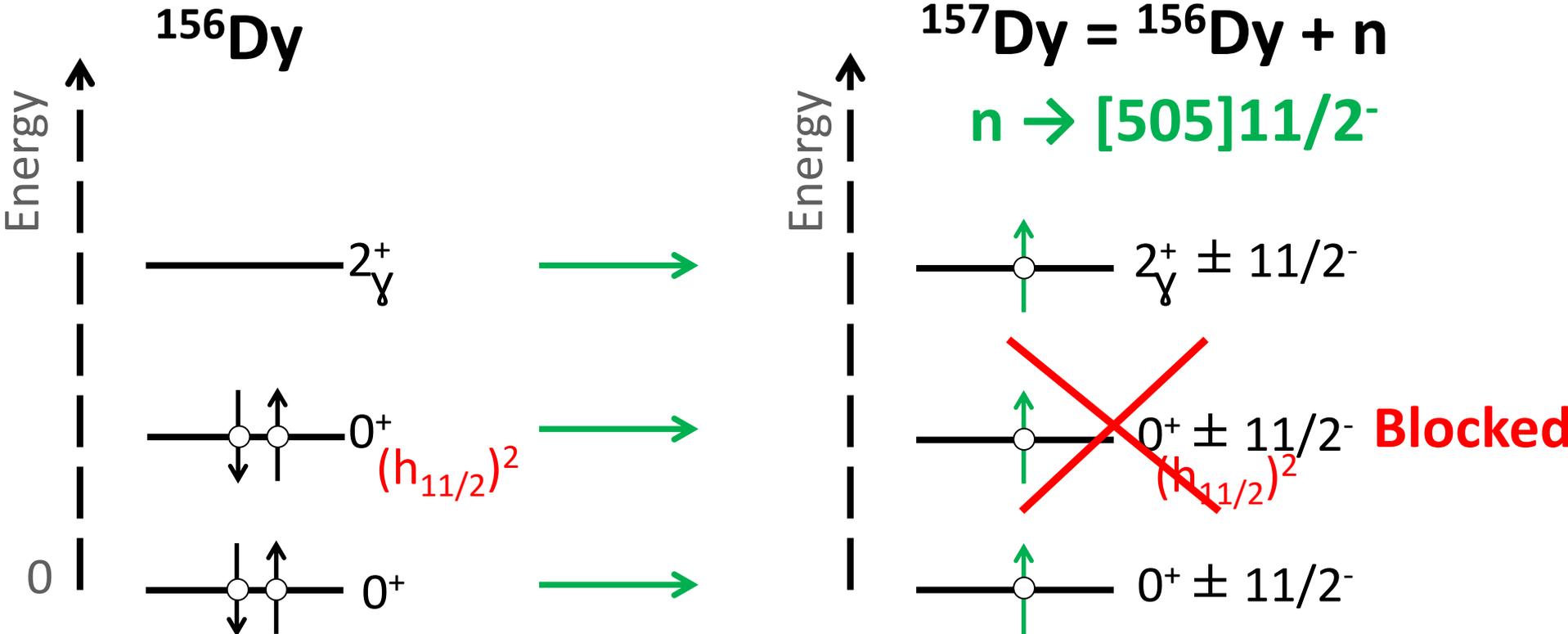
- To get a deeper insight, this study looks at the spectroscopy of ^{156}Dy and ^{157}Dy . The main objective is to observe the coupling of the $h_{11/2}[505]11/2^-$ neutron orbital in ^{157}Dy , to the first-excited $K^\pi = 0^+$ and $K^\pi = 2^+$ rotational bands of ^{156}Dy .



Hypothesis test

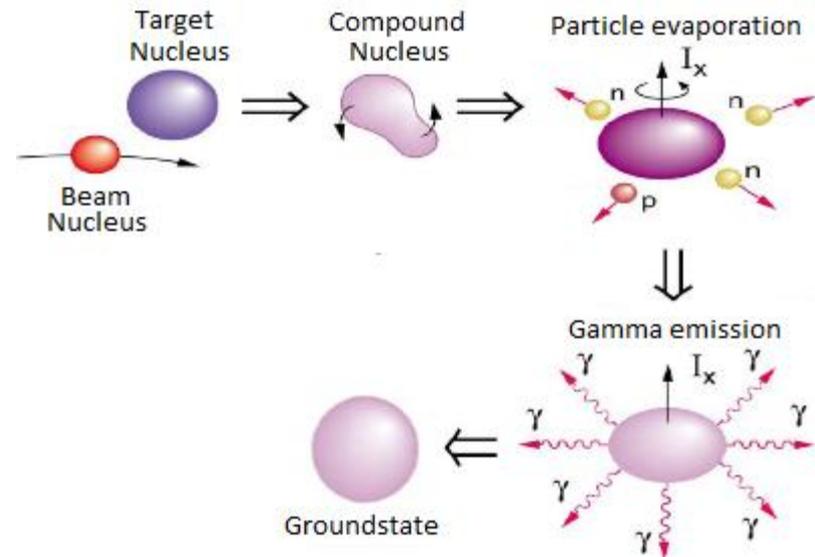


- if the first-excited $K^\pi = 0^+$ state band is comprised of essentially an $h_{11/2}$ pair of neutrons as called for by the quadrupole pairing hypothesis, then the band formed by the coupling of the $h_{11/2}$ neutron orbital to the first-excited $K^\pi = 0^+$ state will be absent or Pauli-blocked in ^{157}Dy , thus providing a yes or no test of the hypothesis.



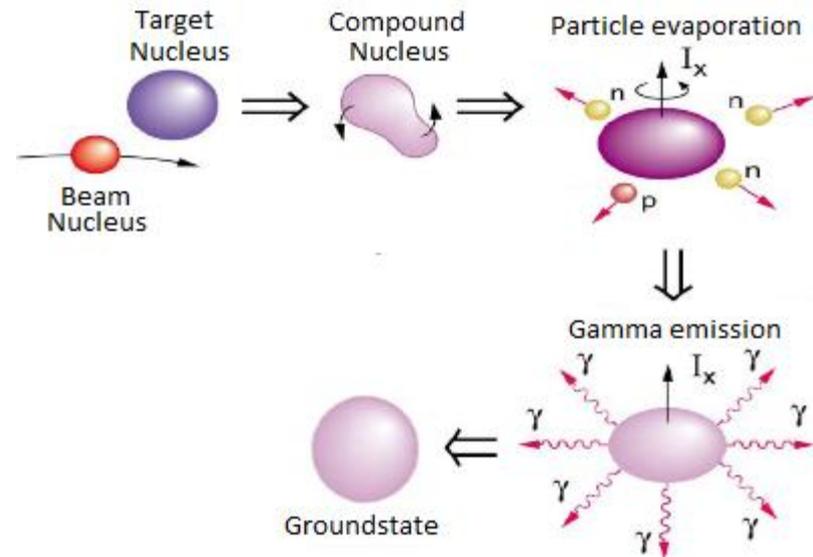
Experimental Details

- $^{155}\text{Gd} (\alpha, 2n) ^{157}\text{Dy}$; 25 MeV \sim (2-3 days)



Experimental Details

- $^{155}\text{Gd} (\alpha, 2n) ^{157}\text{Dy}$; 25 MeV \sim (2-3 days)



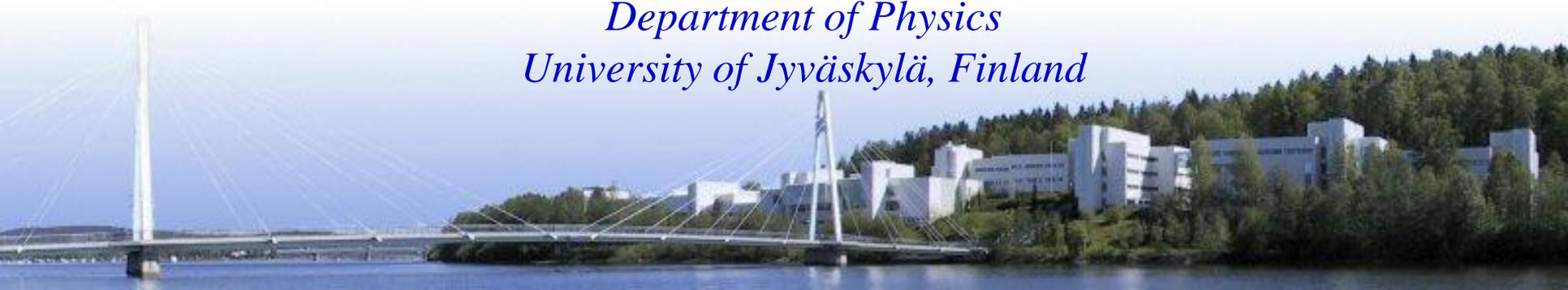
JUROGAM II ARRAY

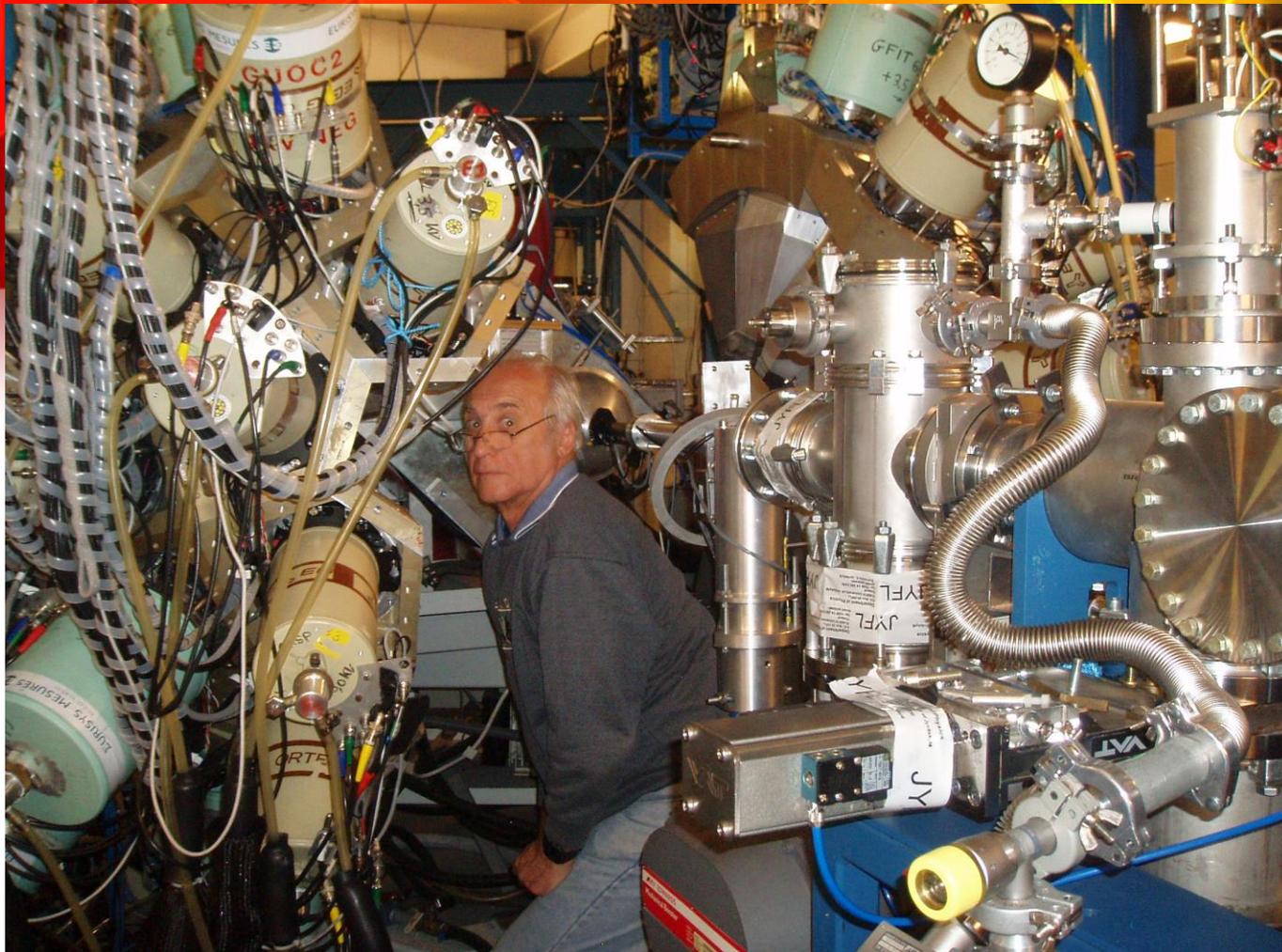


Collaborators



*Department of Physics
University of Jyväskylä, Finland*





Previous in-beam works

Hayakawa et al, Eur, Phys. J. A 15, 299-302 (2002)

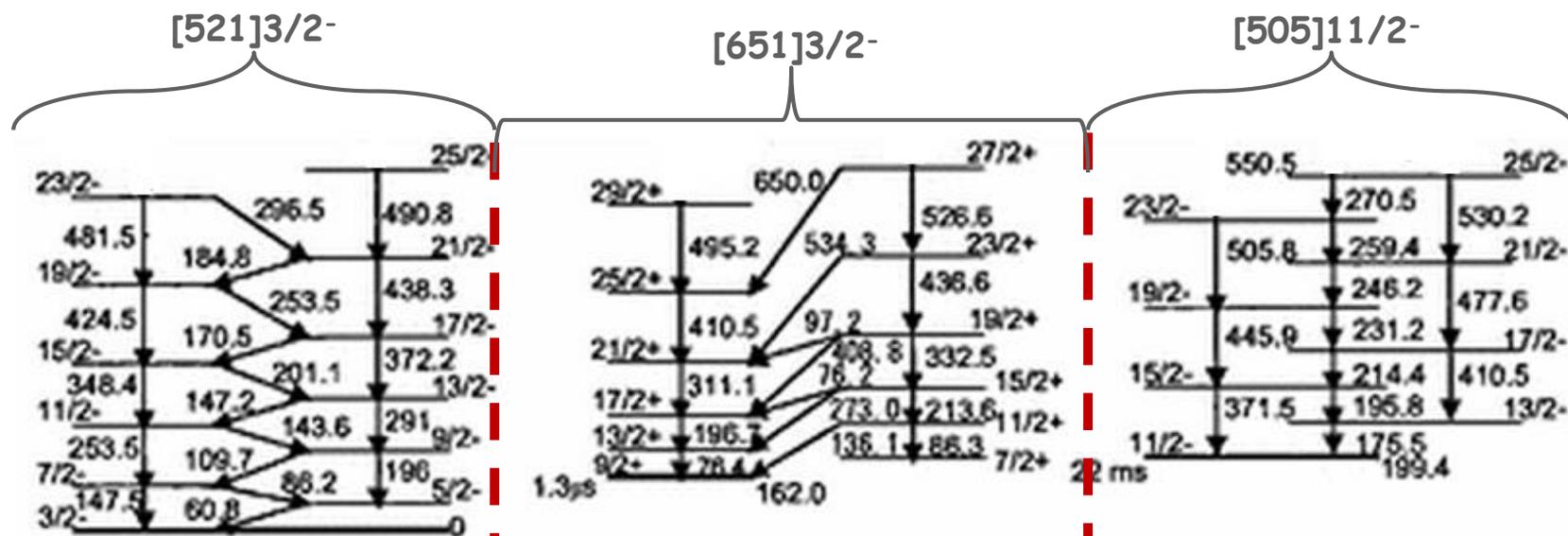
Eur. Phys. J. A 15, 299–302 (2002)
DOI 10.1140/epja/i2002-10082-0

THE EUROPEAN
PHYSICAL JOURNAL A

Short Note

Rotational alignment of the $h_{11/2}$ band in ^{157}Dy

T. Hayakawa^{1,a}, Y. Toh¹, M. Oshima¹, M. Matsuda¹, Y. Hatsukawa¹, J. Katakura¹, H. Iimura¹, T. Shizuma¹, S. Mitarai², M. Sugawara³, H. Kusakari⁴, and Y.H. Zhang⁵



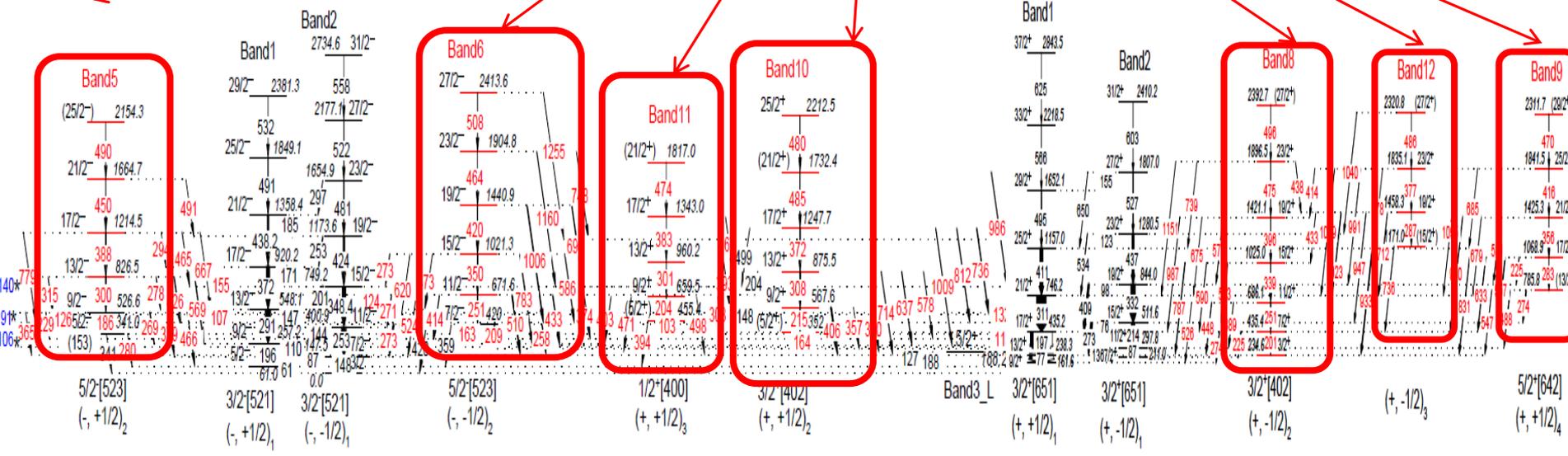
Results & Discussion

- New additions in ^{157}Dy

11 New Bands

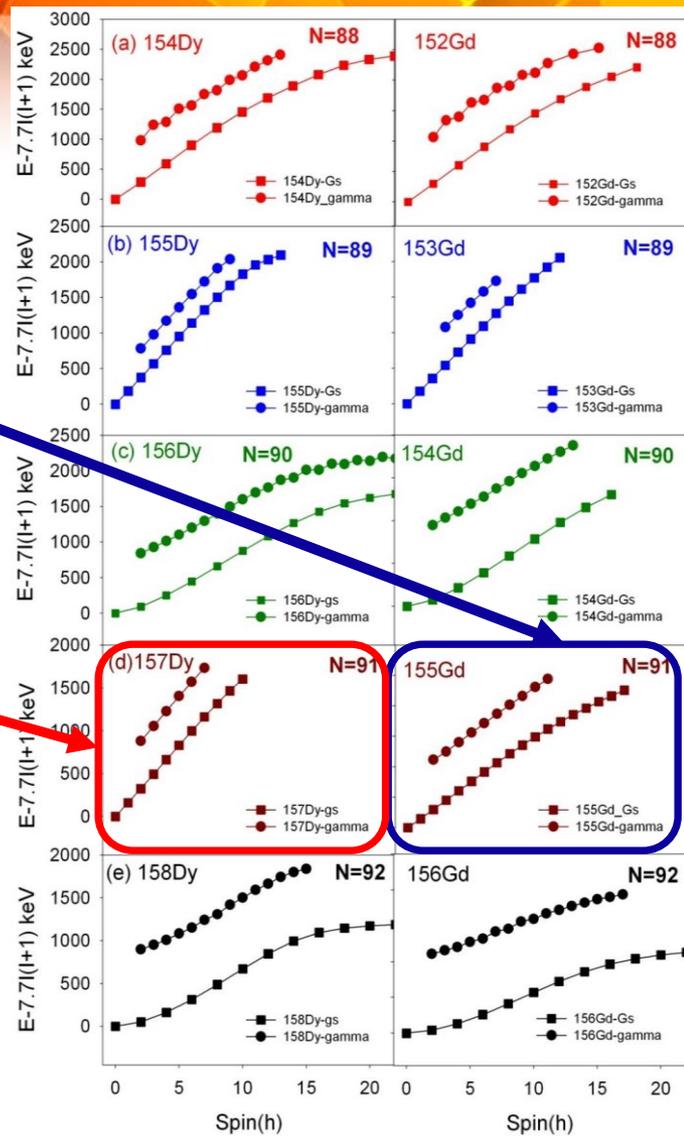
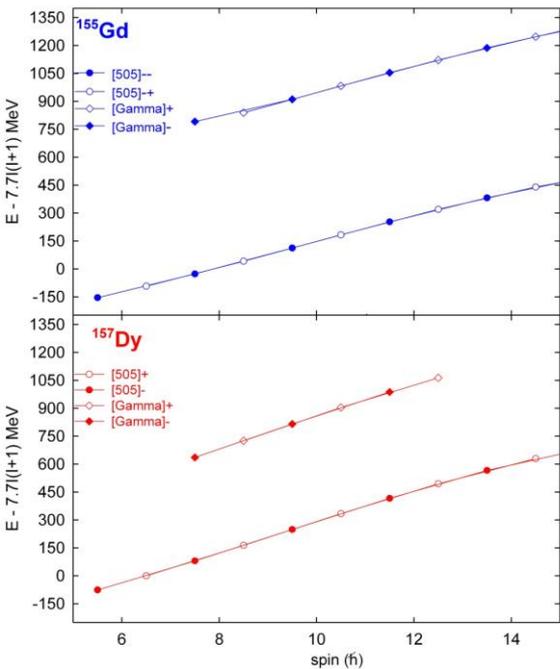
[521]3/2-

[651]3/2-



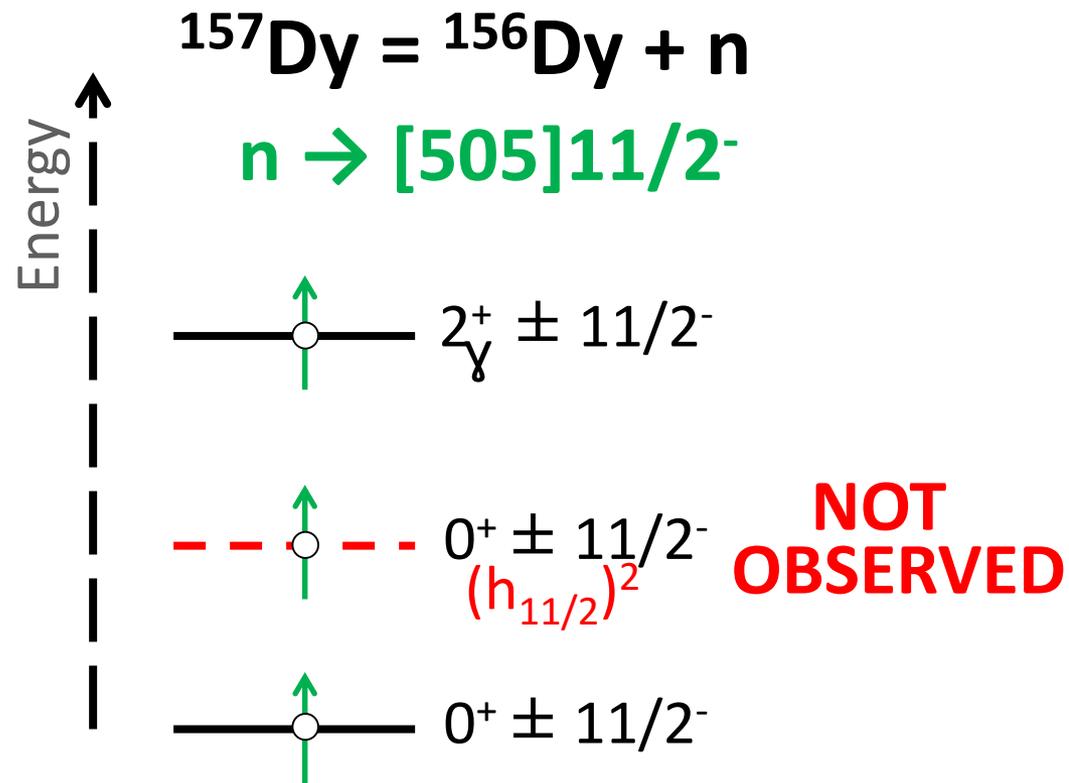
Results: New additions in ^{157}Dy

J. Sharpey-Schafer et al, Eur Phys G



Conclusion

- First γ -bands in the odd-even Dy isotope has been observed
- The Tracking of $[505]11/2^-$ by its own γ -bands demonstrates the nature of γ -bands
- No sign of β -band





End of part 1

Part 2

Systematics of β and γ bands in even-even $N \approx 90$ nuclei

Background and motivation

-Studied by iThemba LABS

This study ranges from Sm to Yb isotopes, with neutron numbers from $N = 88-92$

	152Yb	153Yb	154Yb	155Yb	156Yb	157Yb	158Yb	159Yb	160Yb	161Yb	162Yb	163Yb	164Yb	165Yb	166Yb	167Yb	168Yb
Z	151Tm	152Tm	153Tm	154Tm	155Tm	156Tm	157Tm	158Tm	159Tm	160Tm	161Tm	162Tm	163Tm	164Tm	165Tm	166Tm	167Tm
68	150Er	151Er	152Er	153Er	154Er	155Er	156Er	157Er	158Er	159Er	160Er	161Er	162Er	163Er	164Er	165Er	166Er
	149Ho	150Ho	151Ho	152Ho	153Ho	154Ho	155Ho	156Ho	157Ho	158Ho	159Ho	160Ho	161Ho	162Ho	163Ho	164Ho	165Ho
66	148Dy	149Dy	150Dy	151Dy	152Dy	153Dy	154Dy	155Dy	156Dy	157Dy	158Dy	159Dy	160Dy	161Dy	162Dy	163Dy	164Dy
	147Tb	148Tb	149Tb	150Tb	151Tb	152Tb	153Tb	154Tb	155Tb	156Tb	157Tb	158Tb	159Tb	160Tb	161Tb	162Tb	163Tb
64	146Gd	147Gd	148Gd	149Gd	150Gd	151Gd	152Gd	153Gd	154Gd	155Gd	156Gd	157Gd	158Gd	159Gd	160Gd	161Gd	162Gd
	145Eu	146Eu	147Eu	148Eu	149Eu	150Eu	151Eu	152Eu	153Eu	154Eu	155Eu	156Eu	157Eu	158Eu	159Eu	160Eu	161Eu
62	144Sm	145Sm	146Sm	147Sm	148Sm	149Sm	150Sm	151Sm	152Sm	153Sm	154Sm	155Sm	156Sm	157Sm	158Sm	159Sm	160Sm
	82	84	86	88	90	92	94	96	98	100	102	104	106	108	110	112	N

15 experiments performed to study 12 nuclei

Nucleus (N)	Reaction(s)	Beam energy (MeV)	Target thickness	Statistics events	Spectrometer(s)
¹⁵⁰ Sm (88)	¹³⁶ Xe(¹⁸ O,4n), ¹⁴⁸ Nd(α,2n)	75, 25	~ 5 mg/cm ² 5 mg/cm ²	0.5 × 10 ⁹ γγ 2 × 10 ¹⁰ γγγ	Afrodite, Jurogam II
¹⁵² Gd (88)	¹⁵² Sm (α,4n)	45	5mg/cm ²	0.5 × 10 ⁹ γγ	Afrodite
¹⁵⁴ Dy (88)	¹⁵⁵ Gd(α,4n)	37.5	3.2 mg/cm ²	0.4 × 10 ⁹ γγ	Afrodite
¹⁵⁶ Er (88)	¹⁴⁷ Sm(¹² C,3n)	65	6 mg/cm ²	1.4 × 10 ⁹ γγ	Afrodite
¹⁵⁸ Yb (88)	¹⁴⁴ Sm(¹⁸ O,4n)	78	3 mg/cm ²	2.0 × 10 ⁹ γγ	Afrodite
¹⁵⁴ Gd (90)	¹⁵² Sm(α,2n)	25	4 mg/cm ²	0.5 × 10 ⁹ γγ	Afrodite
¹⁵⁶ Dy (90)	¹⁵⁵ Gd(α,3n), ¹⁴⁸ Nd(¹² C,4n)	25, 65	0.98 mg/cm ² 1.53 mg/cm ²	1.4x10 ⁹ γγ, 2.0 × 10 ⁹ γγγ	Jurogam II, Gammasphere
¹⁵⁸ Er (90)	¹⁵⁰ Sm(¹² C,4n)	65	1 mg/cm ²	4.2 × 10 ⁸ γγ	Afrodite
¹⁶⁰ Yb (90)	¹⁴⁷ Sm(¹⁶ O,3n)	73	4 mg/cm ²	2.0 × 10 ⁹ γγ	Afrodite
¹⁵⁸ Dy (92)	¹⁵⁶ Gd(α,2n), ¹⁵⁵ Gd(α,n)	27, 25	11 mg/cm ² , 0.98 mg/cm ²	1.1 × 10 ⁹ γγ, 1.4x10 ⁹ γγ	Afrodite, Jurogam II
¹⁶⁰ Er (92)	¹⁵² Sm(¹² C,4n)	64	5 mg/cm ²	2.7 × 10 ⁹ γγ	Afrodite

$^{148}\text{Nd}(^{12}\text{C},4n)^{156}\text{Dy}$, Gammasphere

Siyabonga Majola, Darryl Hartley, Lee Riedinger

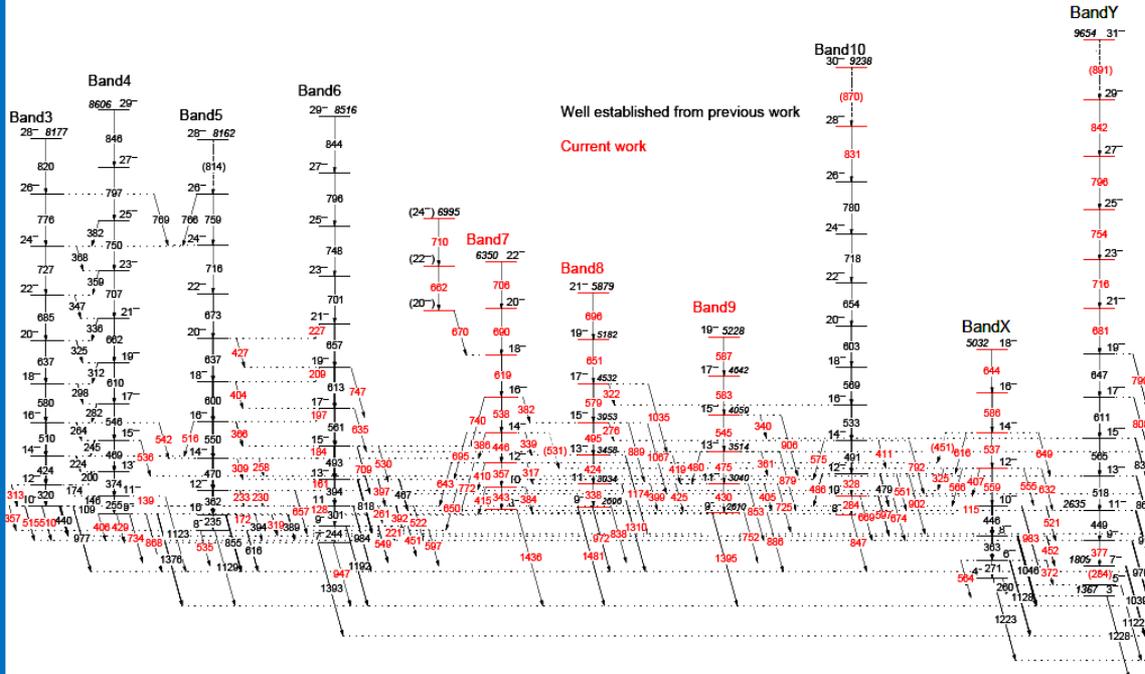


Figure 4.6: Partial level scheme of the negative parity bands deduced on this work for the nucleus of interest. Levels and transitions colored in red represent new findings from this work. On the other hand, everything colored in black indicate findings from previous studies.

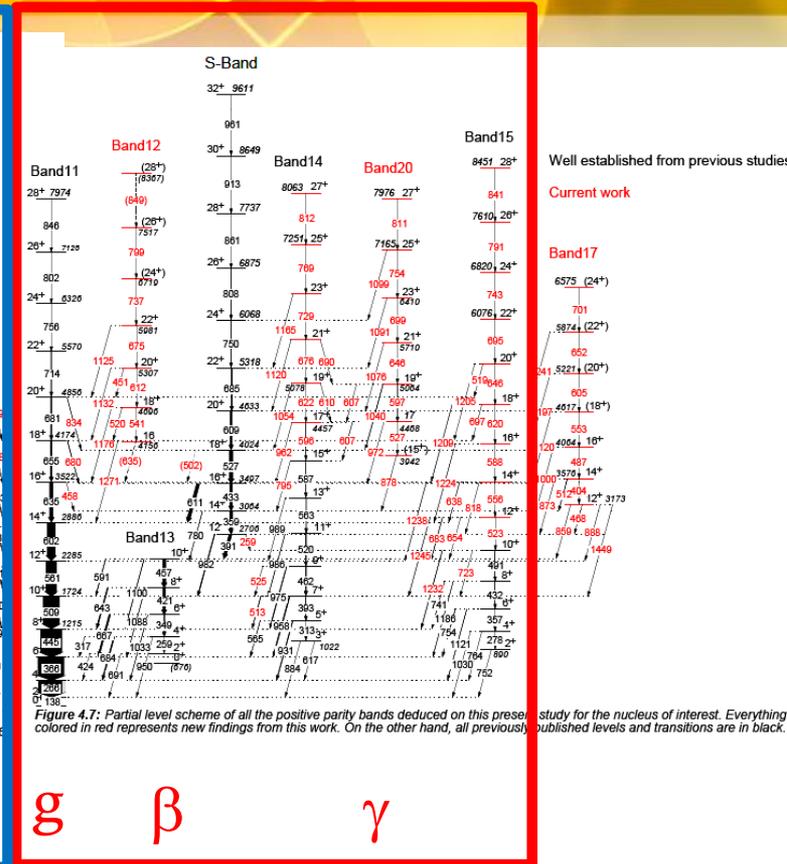


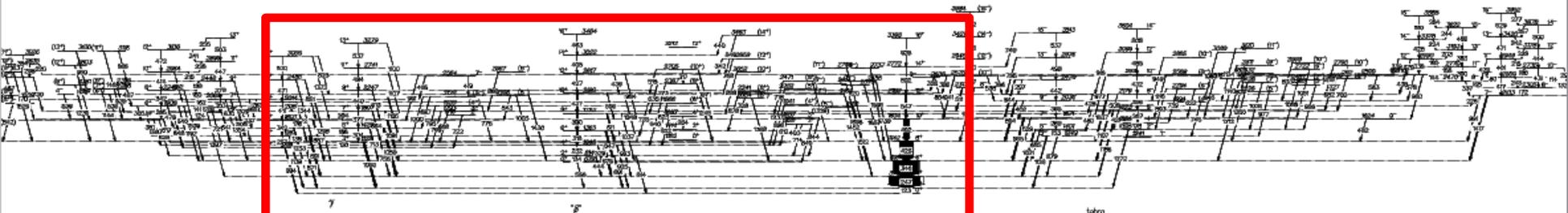
Figure 4.7: Partial level scheme of all the positive parity bands deduced on this present study for the nucleus of interest. Everything colored in red represents new findings from this work. On the other hand, all previously published levels and transitions are in black.

g β γ

S. Majola, D. Hartley, L. Riedinger, et al PRC 91 034330 (2015)
 D. Hartley, L. Riedinger, S. Majola, et al PRC 95 014321 (2017)

$^{152}\text{Sm}(\alpha, 2n)^{154}\text{Gd}$

^{154}Gd JF Sharpey-Shafer et al

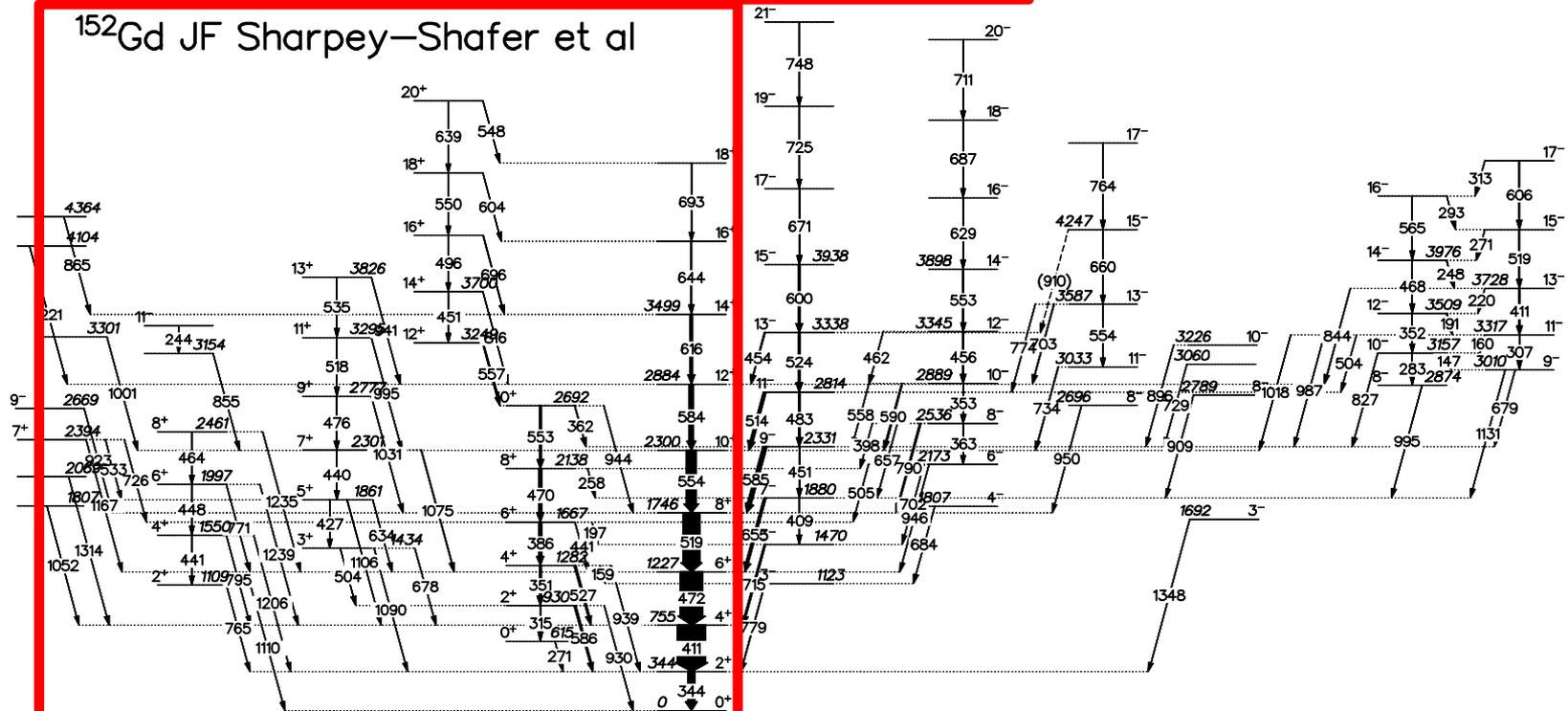


γ

β

g

^{152}Gd JF Sharpey-Shafer et al



γ

β

g

tetra?

γ

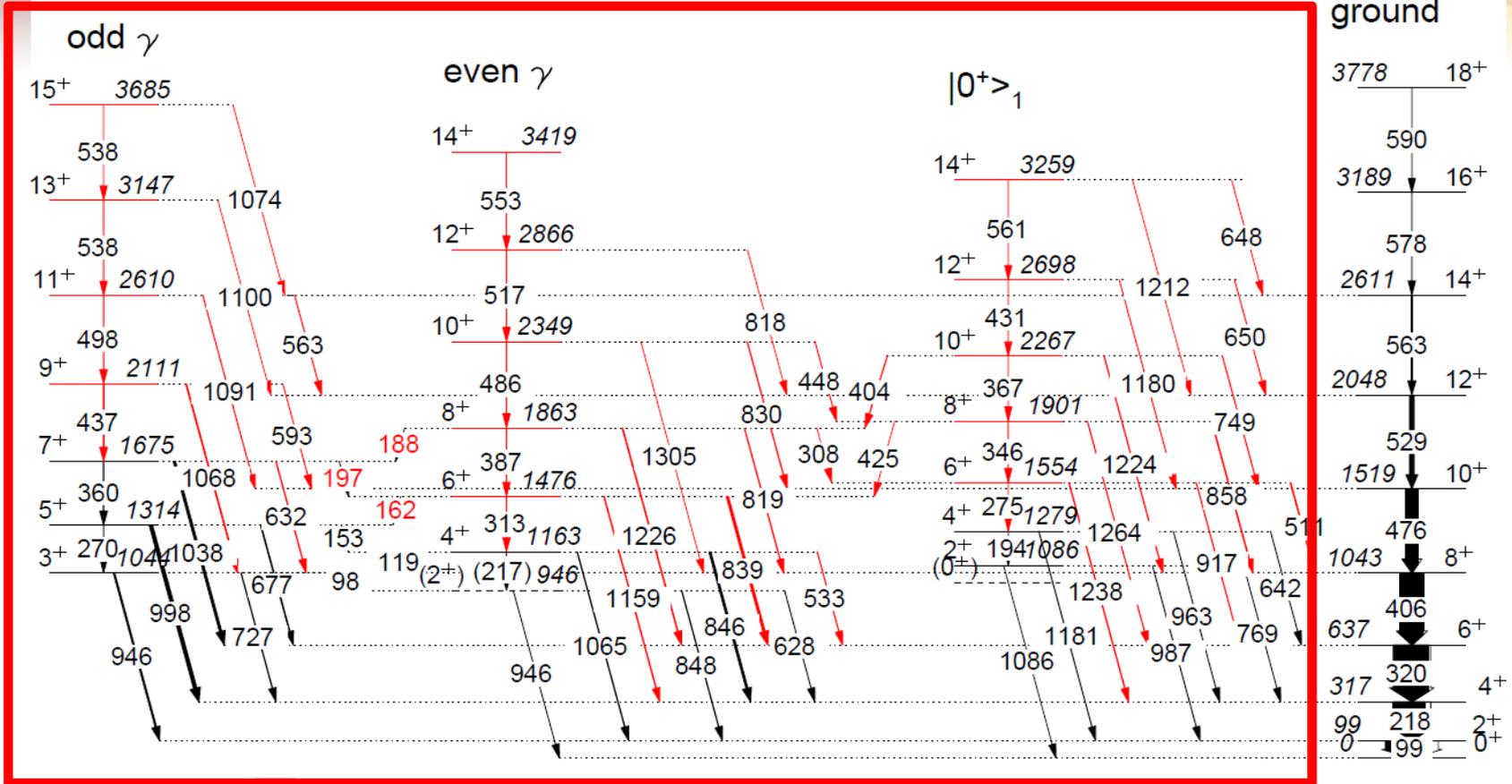
β

g

$^{156}\text{Gd}(\alpha, 2n) ^{158}\text{Dy}$, AFRODITE

^{158}Dy (N=92)

Maciek Stankiewicz



S. Majola, et al., PRC, to be published

Beta & Gamma Bands identified to ~12h for the first time in many cases

Also:

^{156}Er JM Rees et al, PRC83, 044314(11)

ES Paul et al, PRC79 044324(09)

^{160}Er K Dushing et al. PRC73 014317(06)

J Ollier et al, PRC, 064322 (09)

^{158}Dy T Hayakwaw PRC 68, 067303(03)

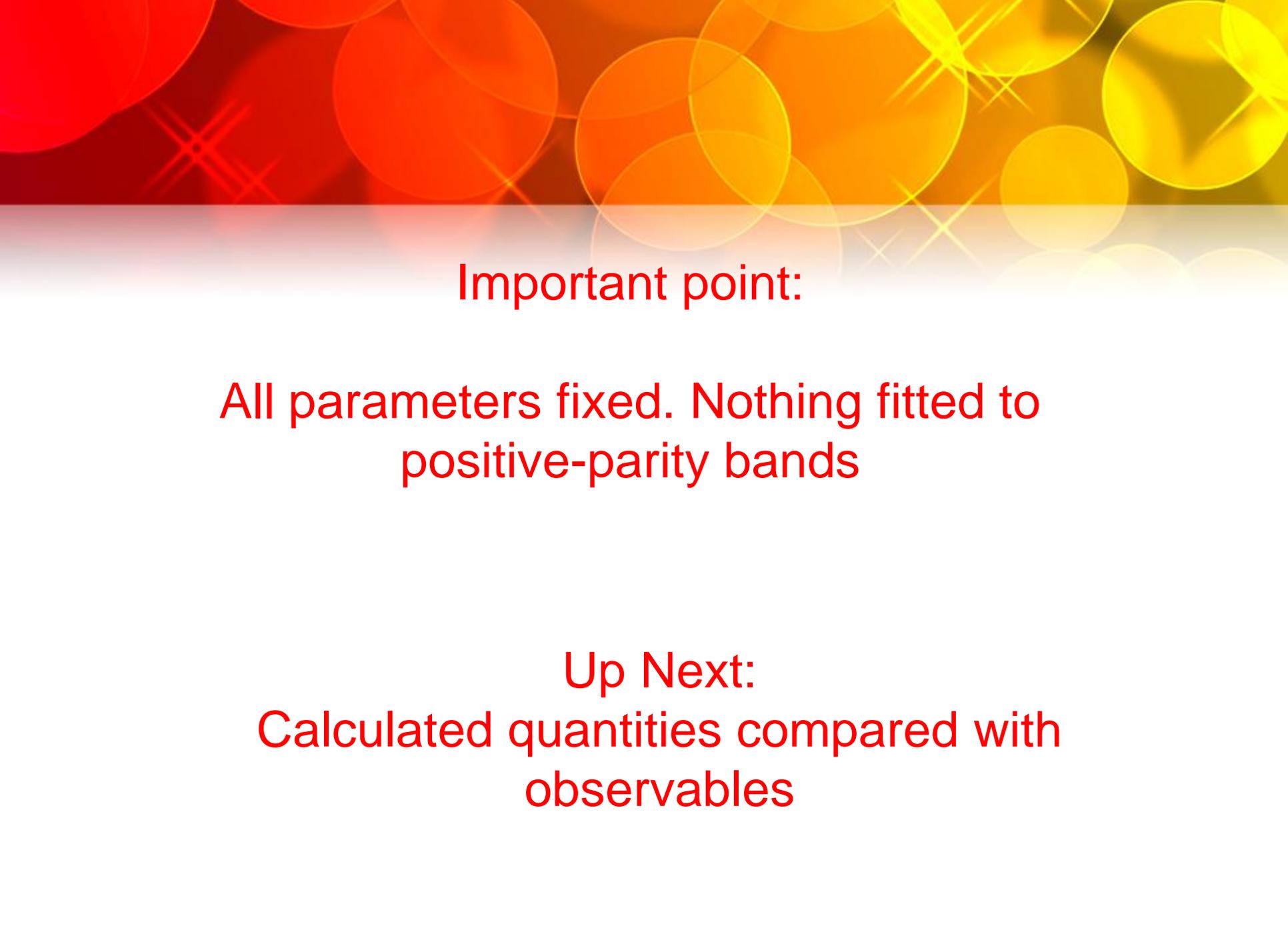
^{152}Sm P Garrett et al PRL 103, 062501(09)

Relativistic Mean Field “Density Functionals” Coupled
to the:

Bohr Hamiltonian

Zhipan Li
Jie Meng
Chunyan Song
Jiangjing Yao
Shuangquan Zhang

Zhi Shi
Bangyan Song



Important point:

All parameters fixed. Nothing fitted to
positive-parity bands

Up Next:
Calculated quantities compared with
observables

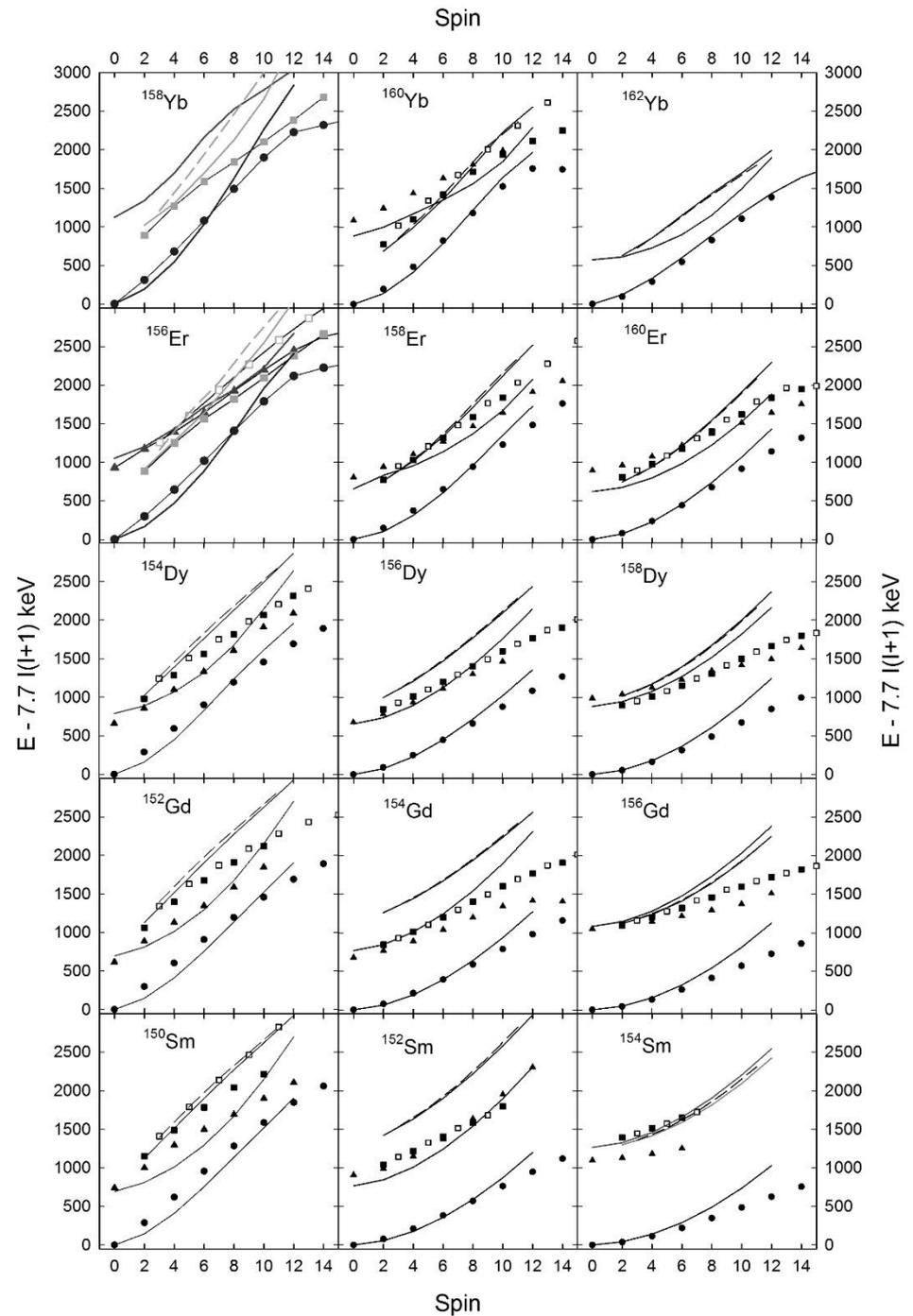
Beta & Gamma Bands identified to $\sim 12\text{h}$ for the first time in many cases

The determination of a comprehensive set of level energies and branching ratios **between bands allows their** electromagnetic properties to be compared to nuclear models. To come to an understanding of the properties of these bands, **the data obtained in this work are explained** using a relativistic mean-field combined with the Bohr Hamiltonian.

Calculated Energies
(lines)
compared to experiment
(points)

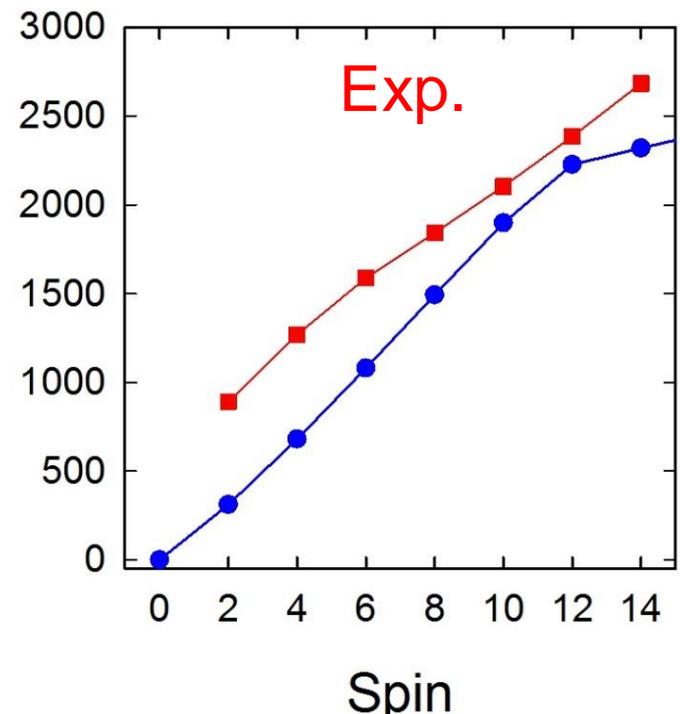
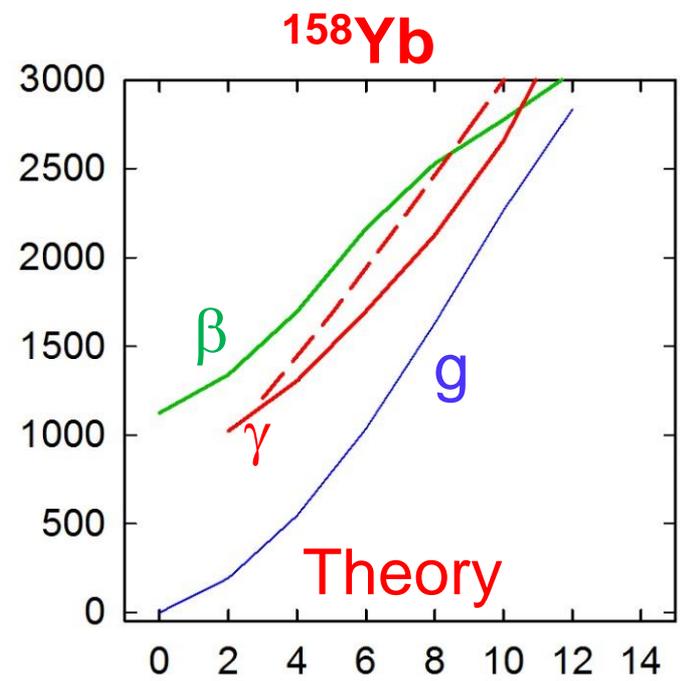


ground band
 β band
 γ band



N = 88

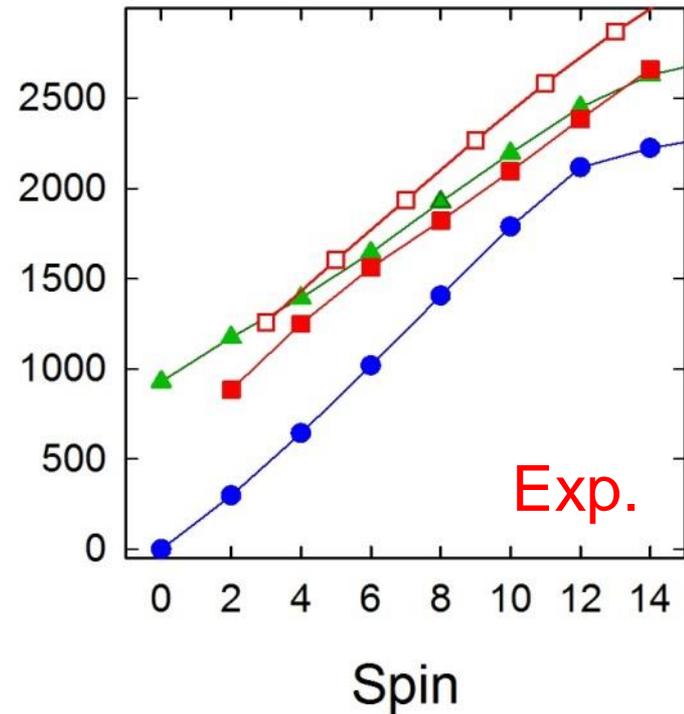
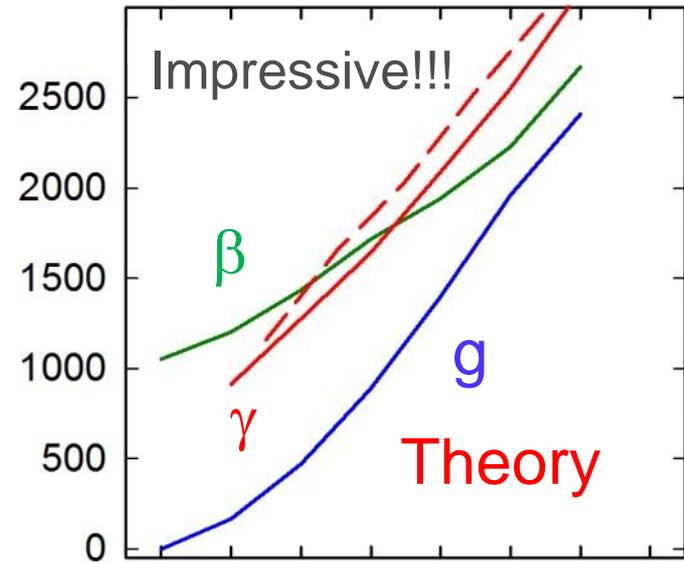
E - 7.7 I(I+1) keV



N = 88

- predicts different moments of inertia for β
and γ bands

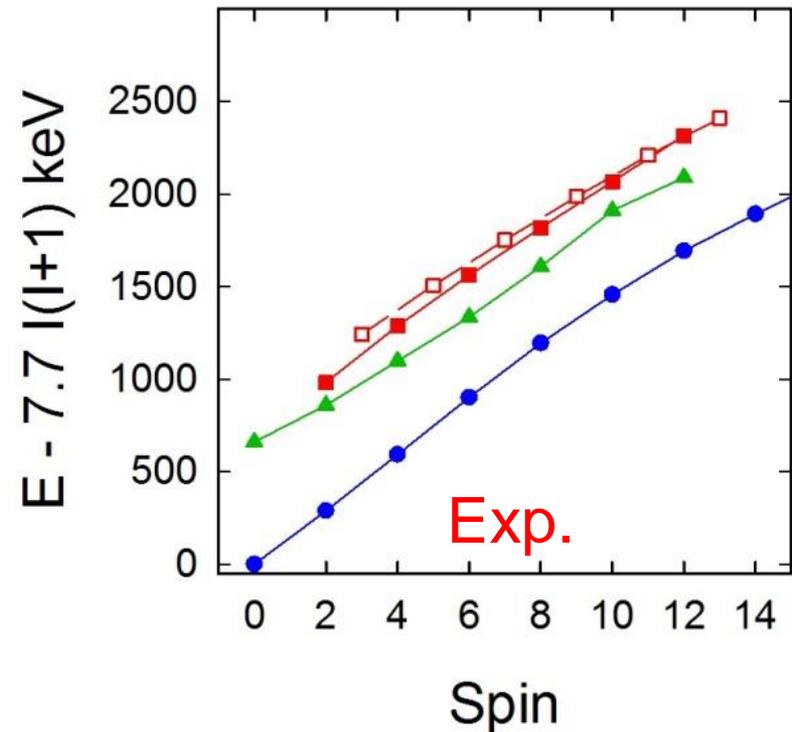
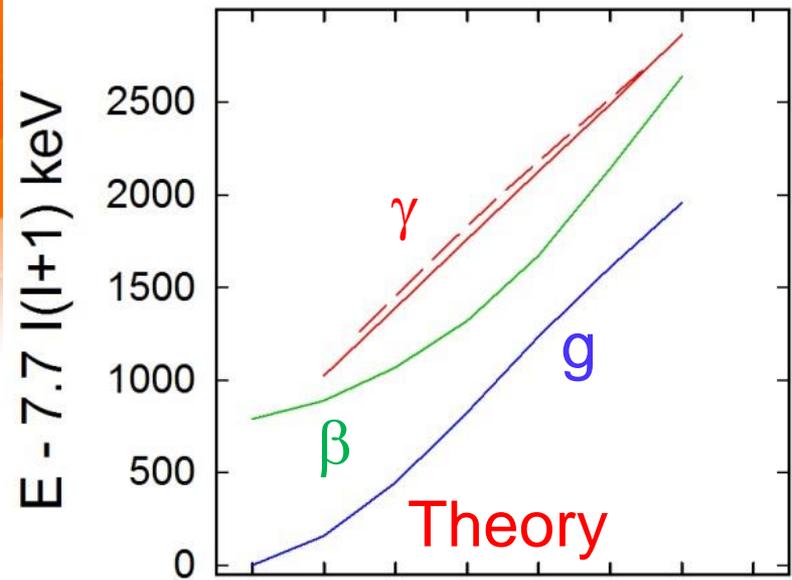
E - 7.7 I(I+1) keV



N = 88

- Predicts the order very well
- Predicts signature splitting very well
 - Ground parallel to γ bands

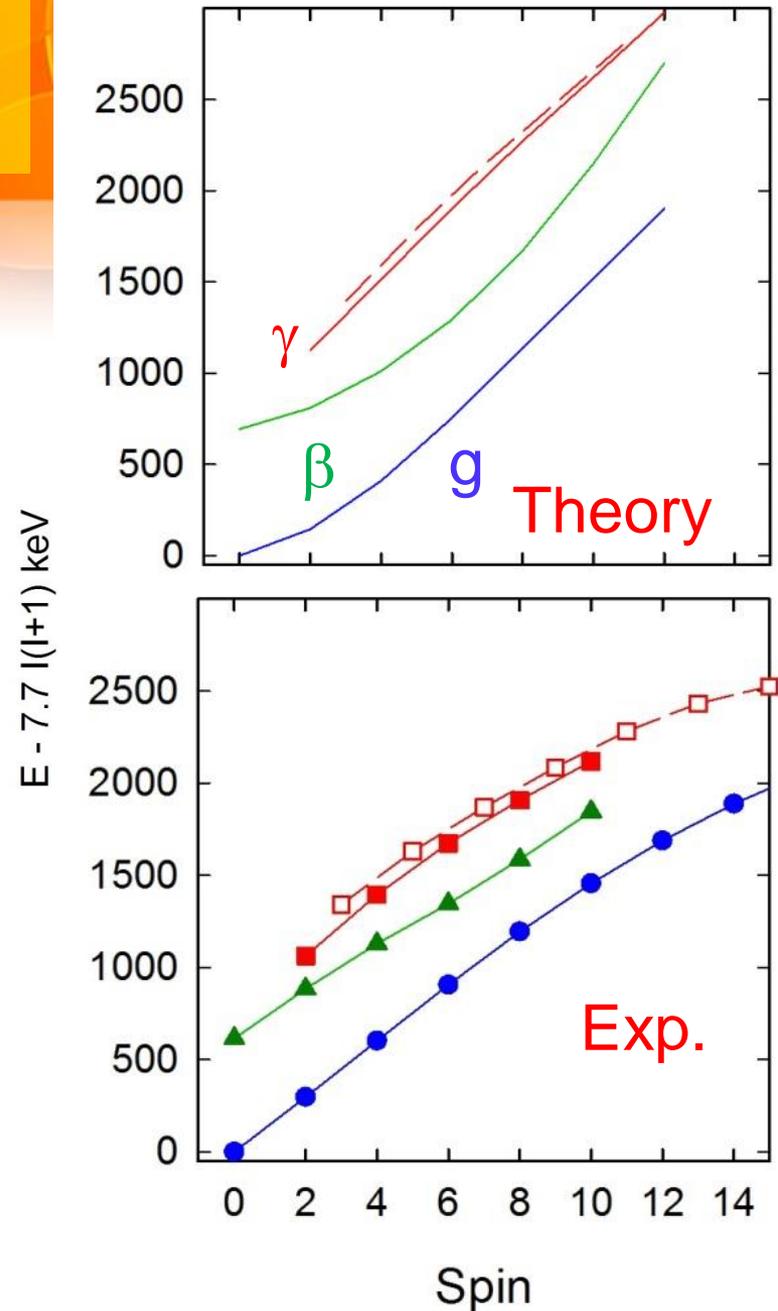
^{154}Dy



N = 88

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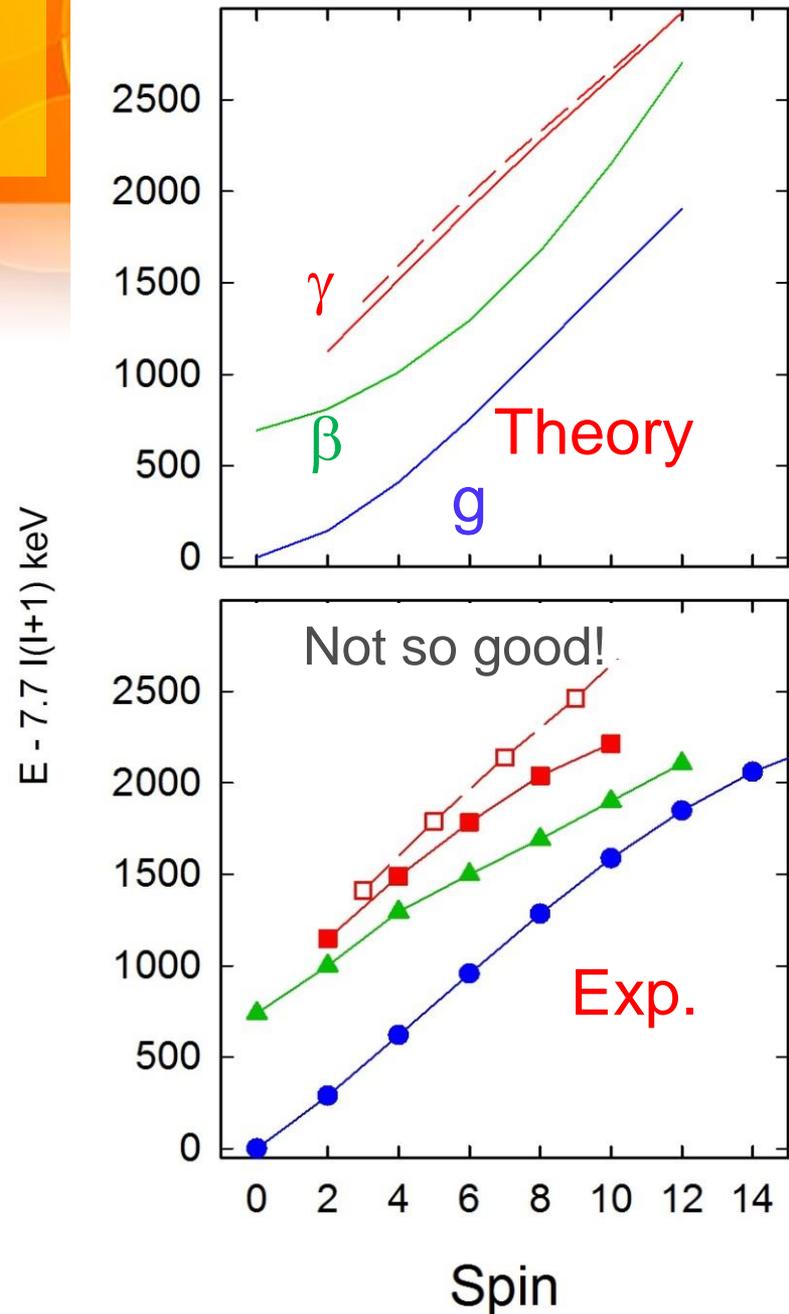
^{152}Gd



N = 88

- N= 88 covered extremely well by the theory

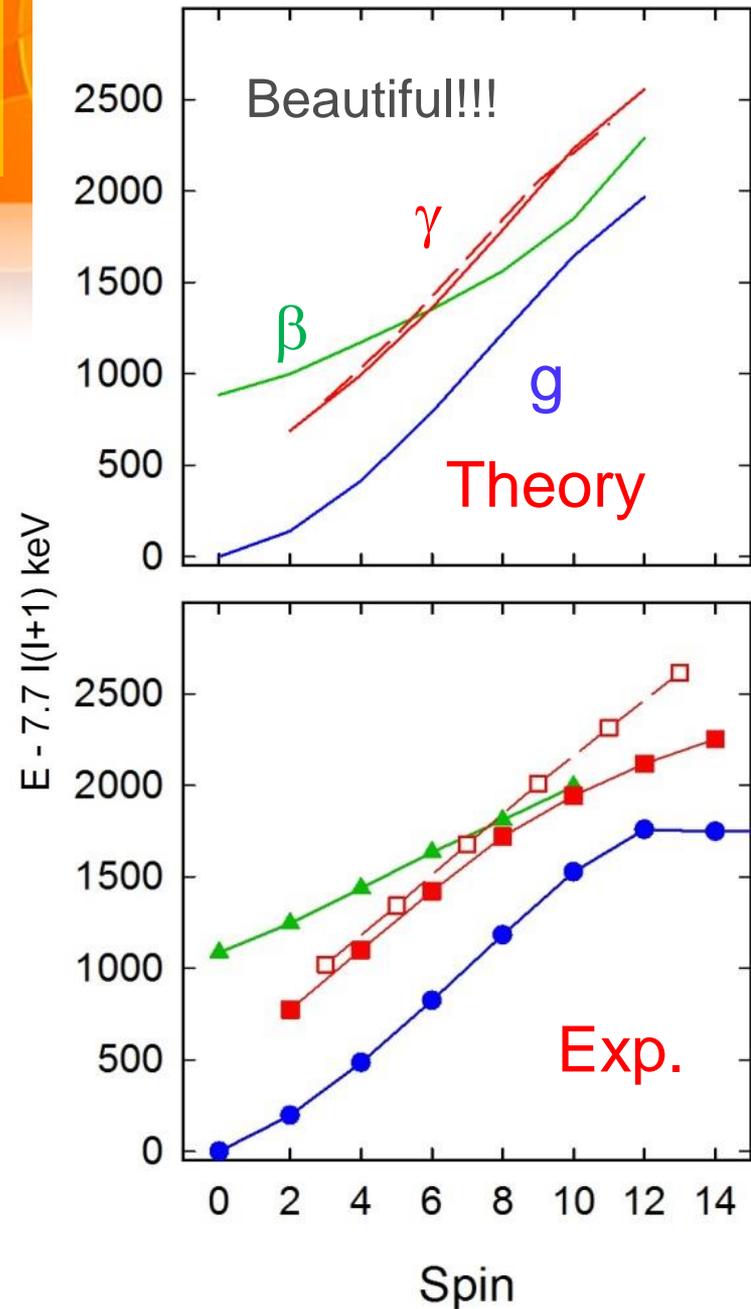
^{150}Sm



N = 90

- predicts different moments of inertia for β and γ bands
- Predicts the order very well
- Ground parallel to γ bands

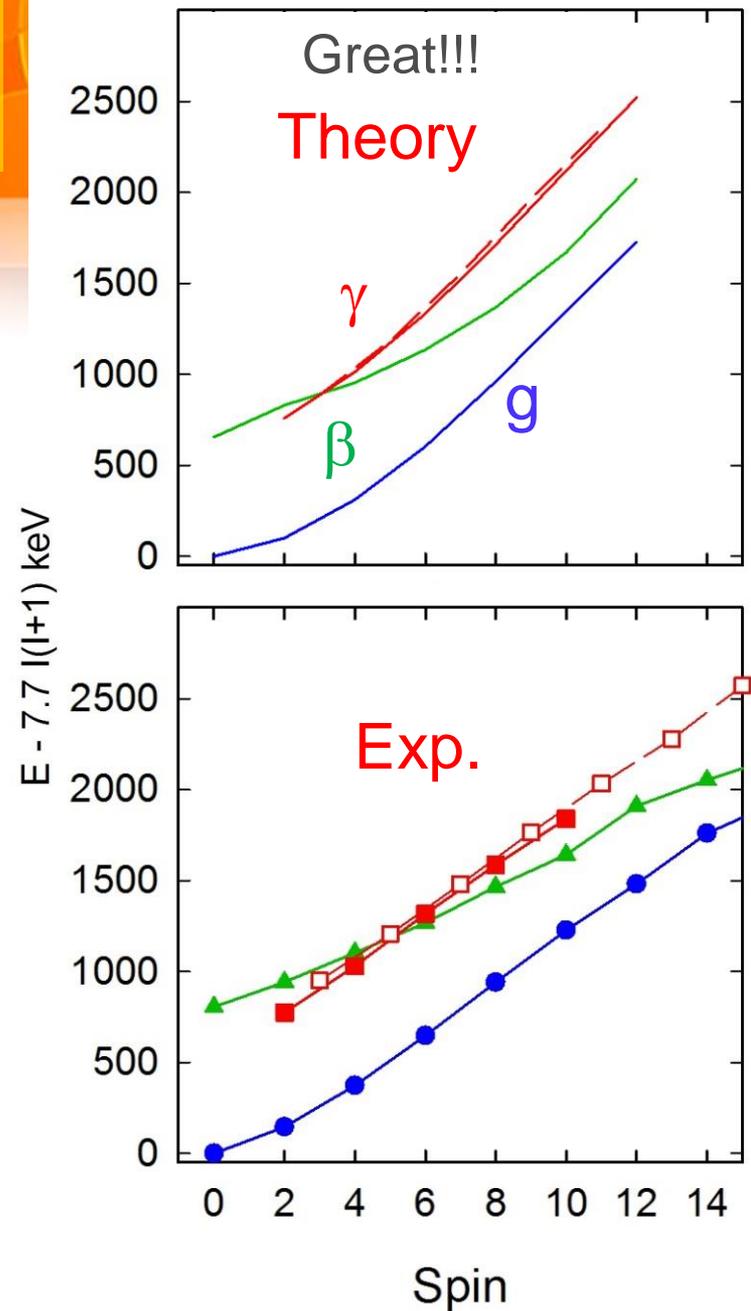
^{160}Yb



N = 90

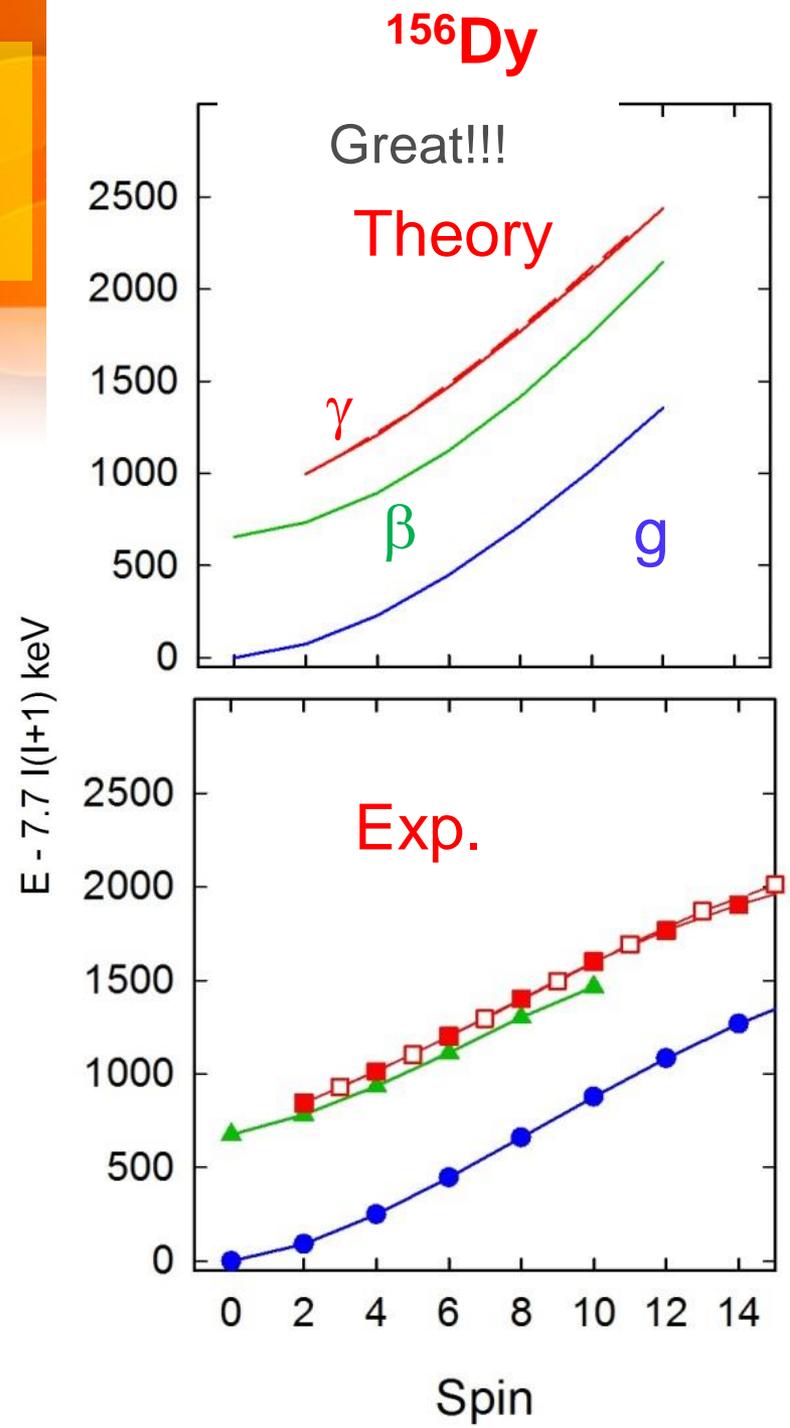
- Same story here

^{158}Er



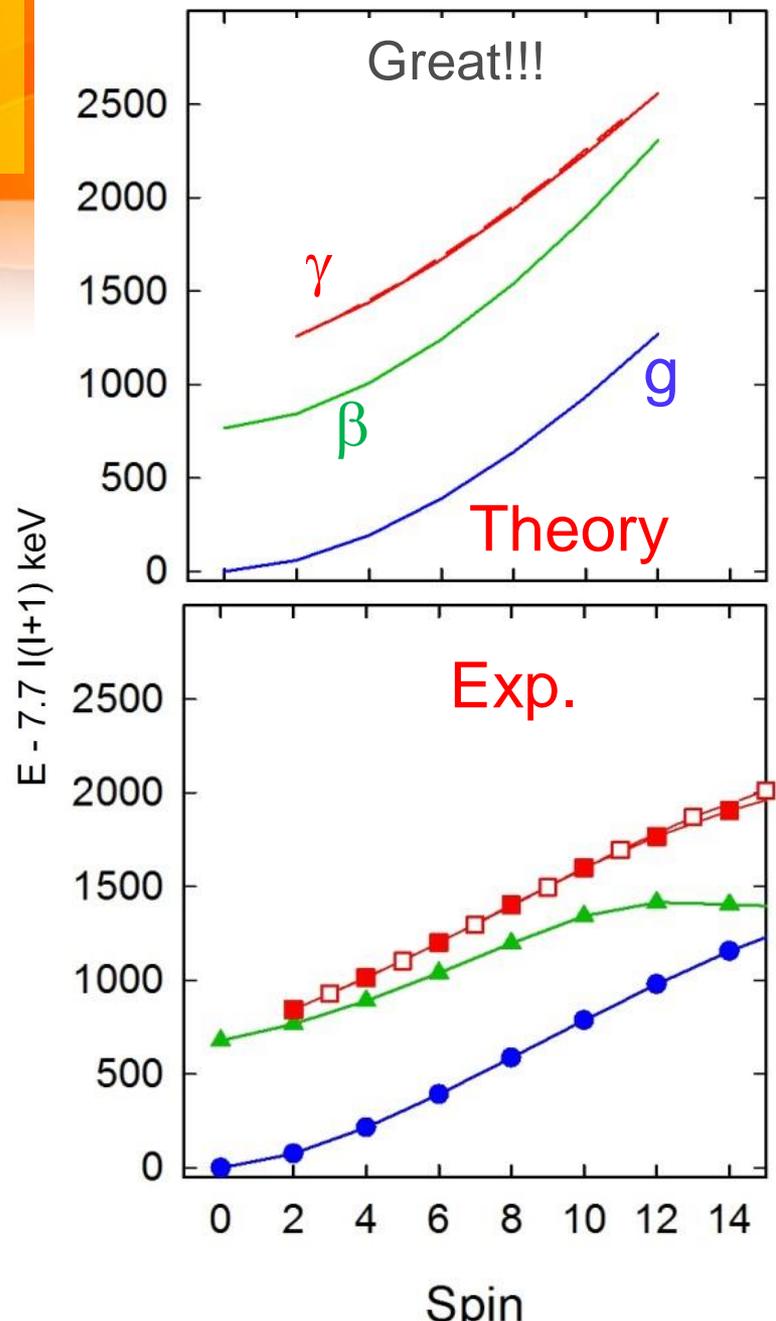
N = 90

- predicts similar moments of inertia for β and γ bands
 - Predicts the order very well
 - Again! ground parallel to γ bands



N = 90

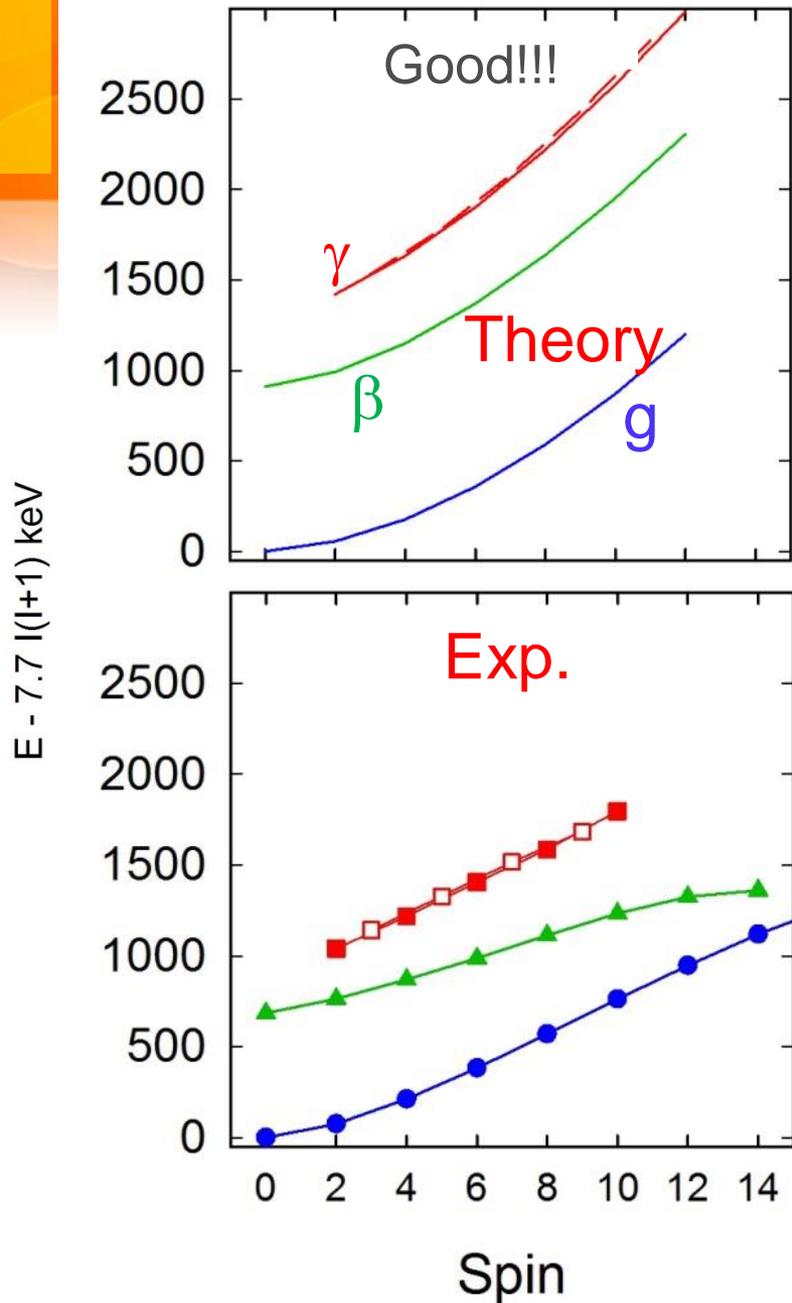
^{154}Gd



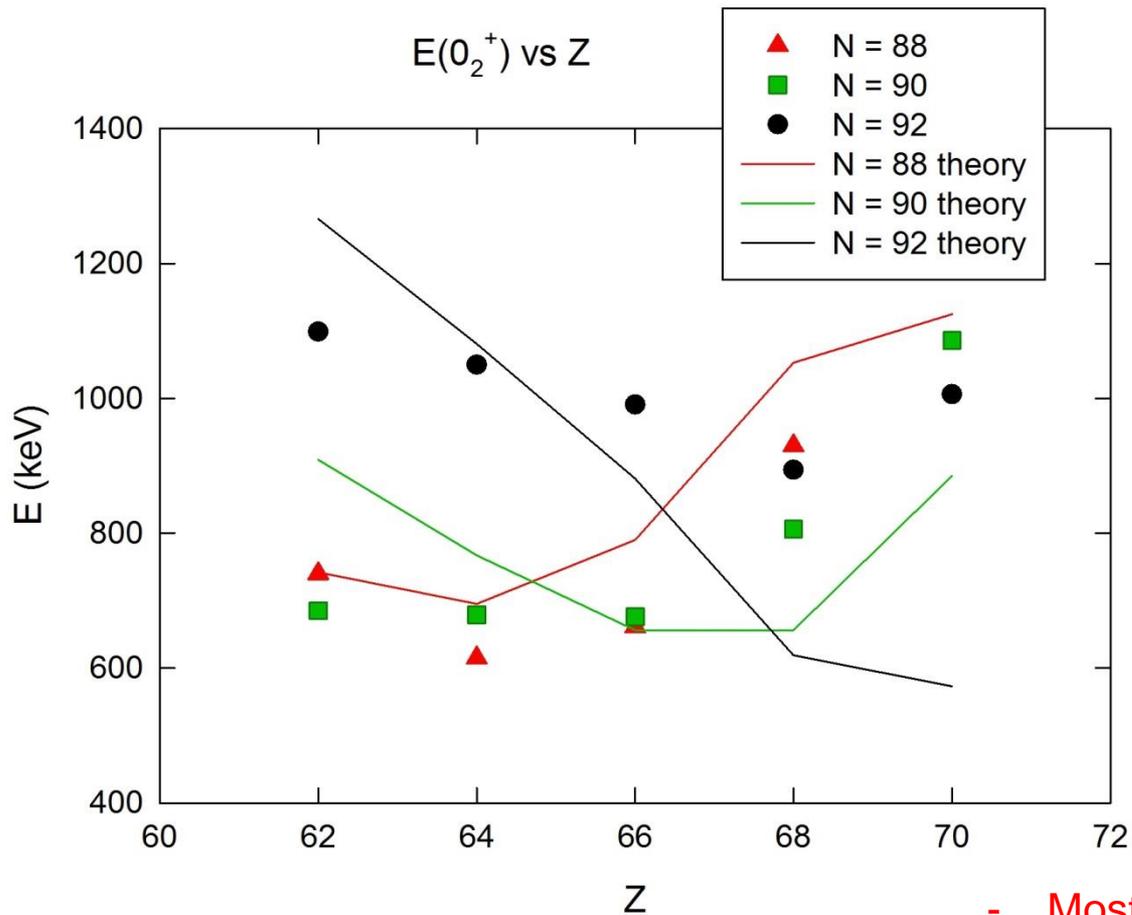
N = 90

- N= 90 covered extremely well by the theory

^{152}Sm

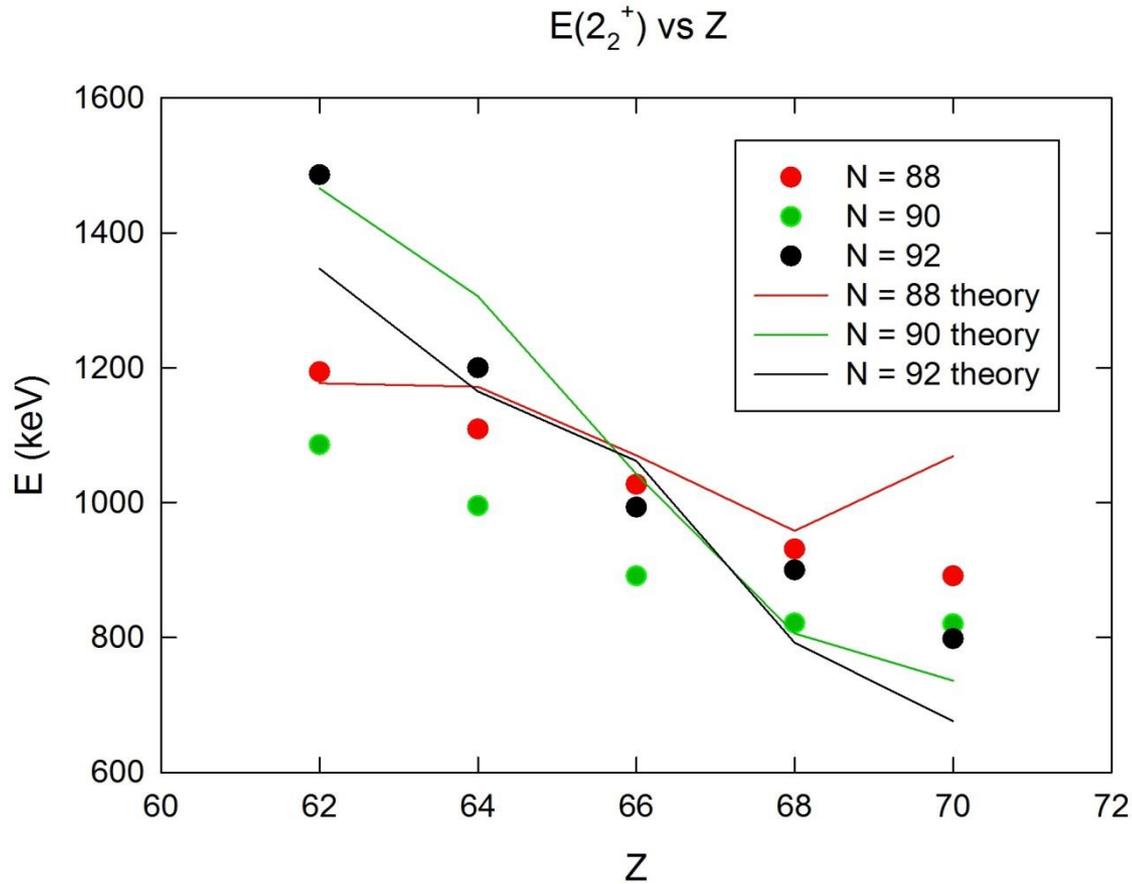


Bandhead energies: O_2^+ states



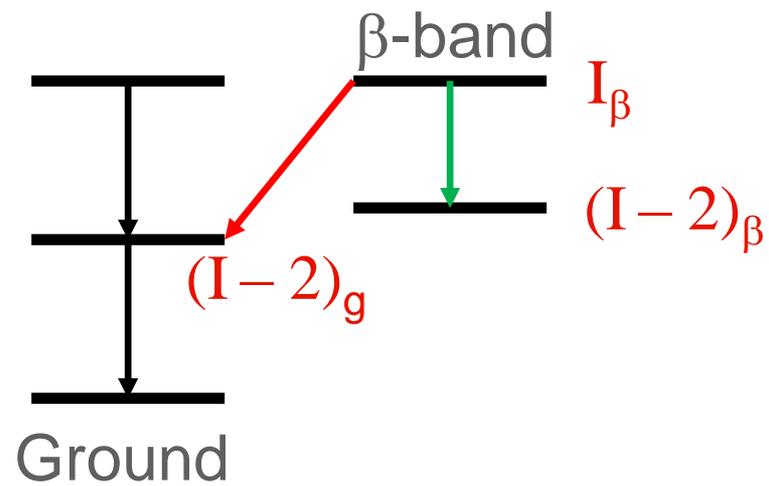
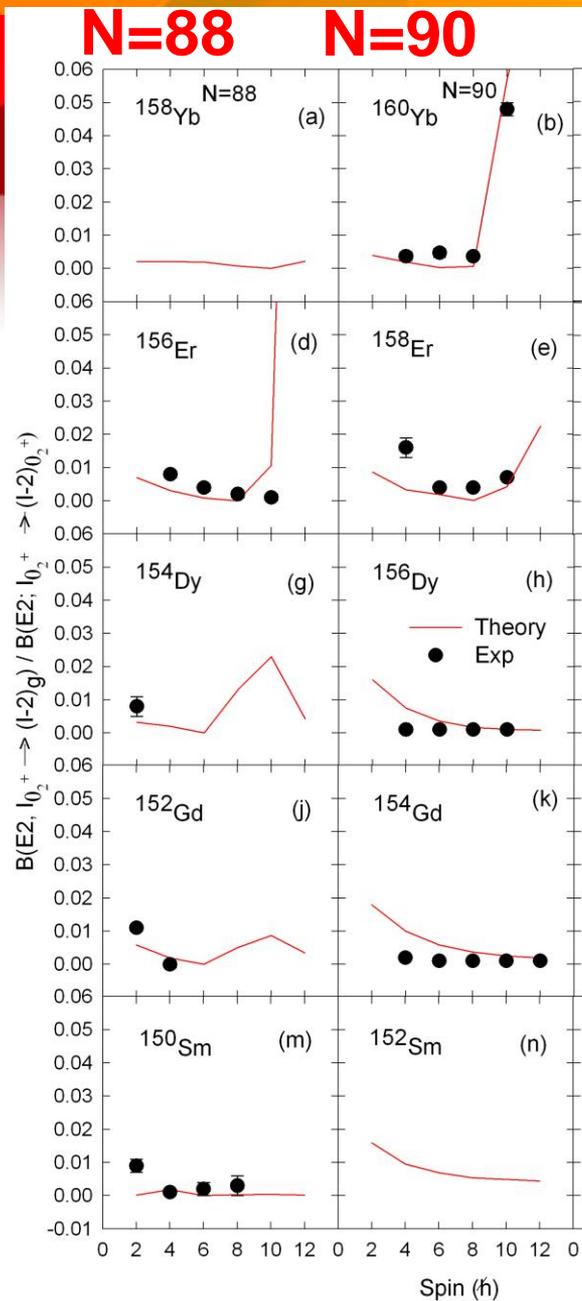
- Most N = 92 underestimated

Bandhead energies: 2_2^+ states

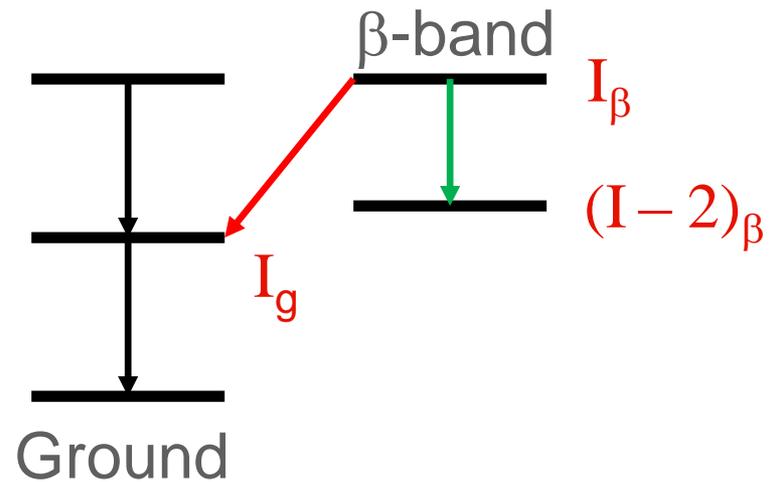
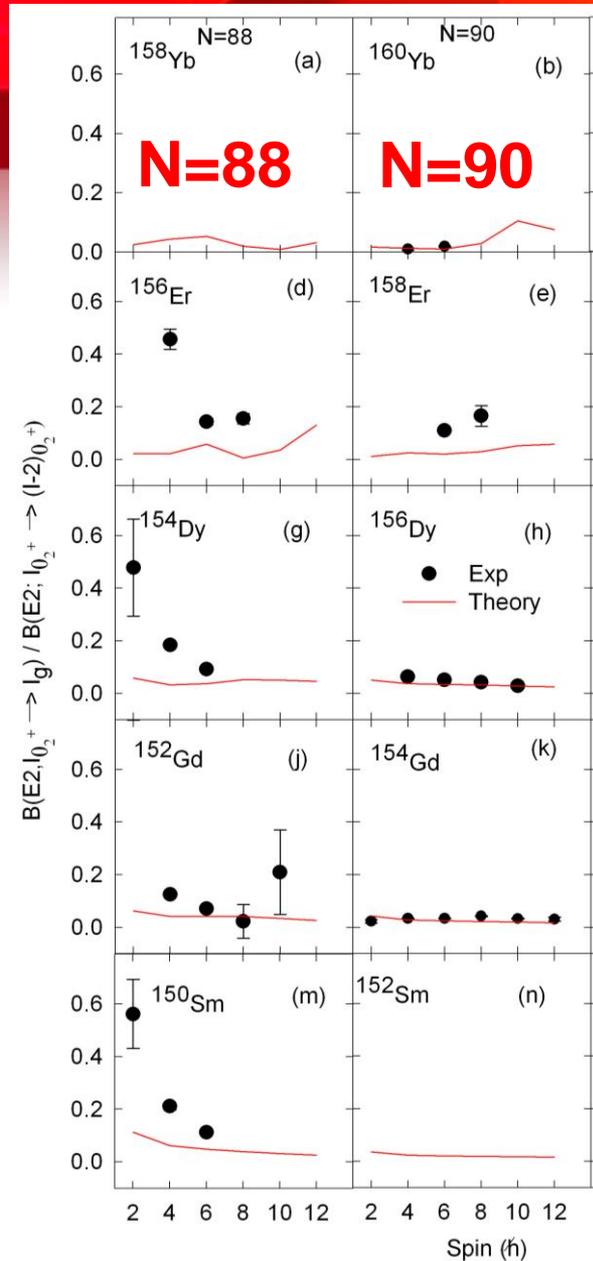


- Trends are generally good

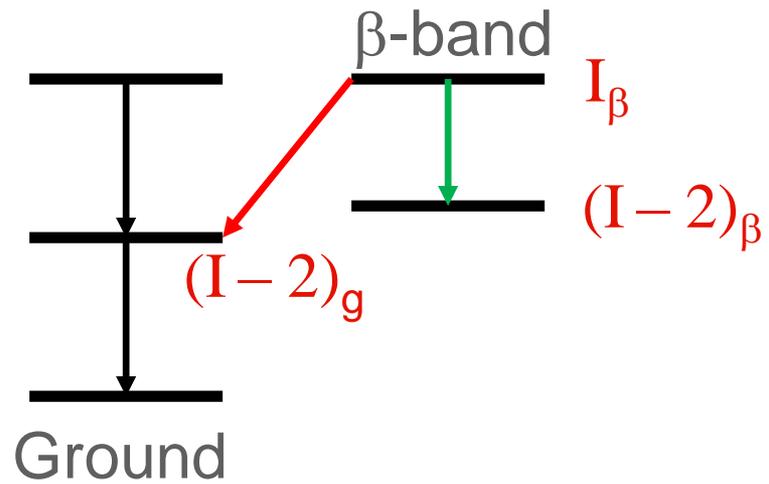
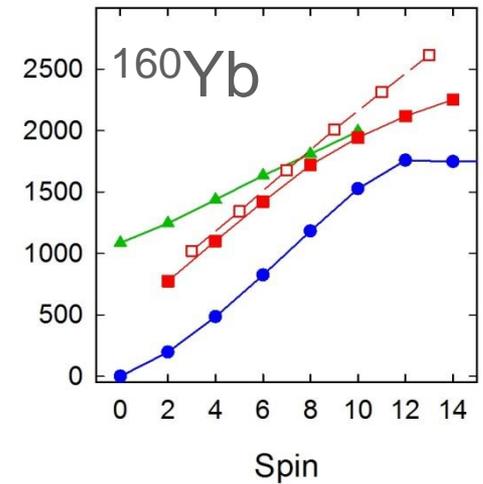
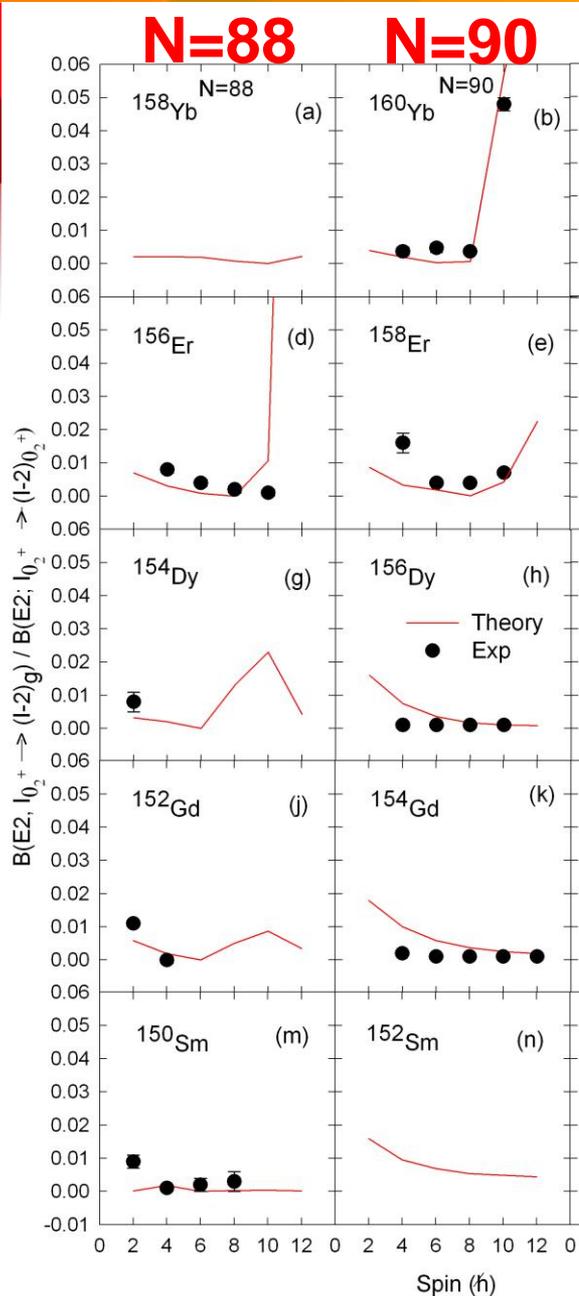
Branching Ratios β -band



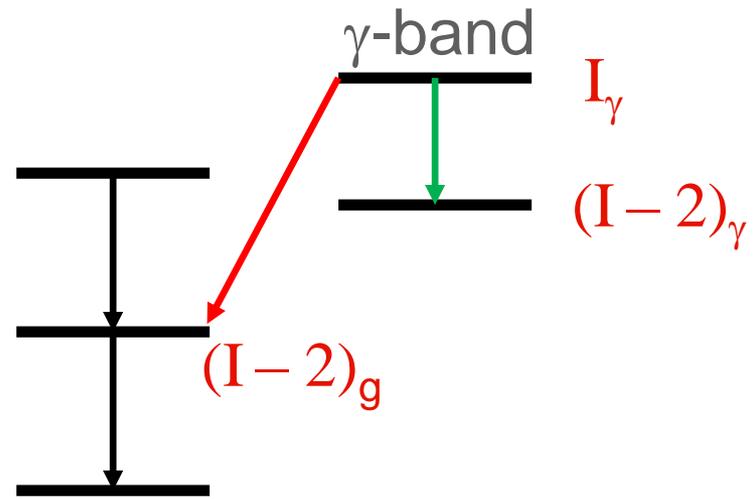
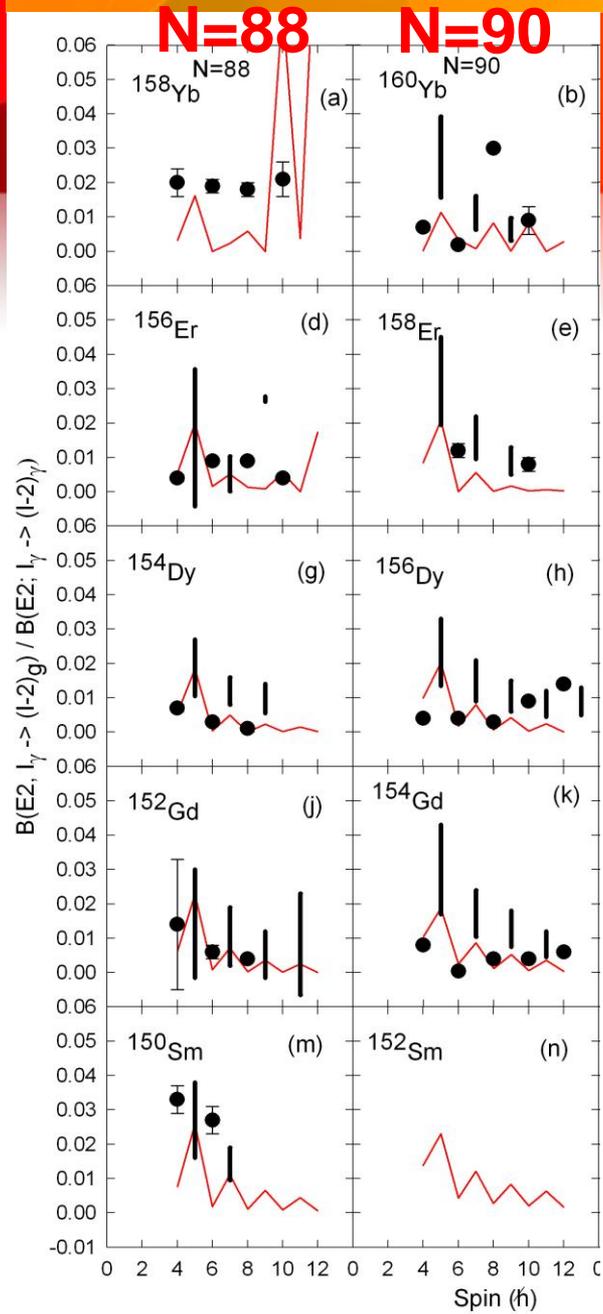
Branching Ratios β -band



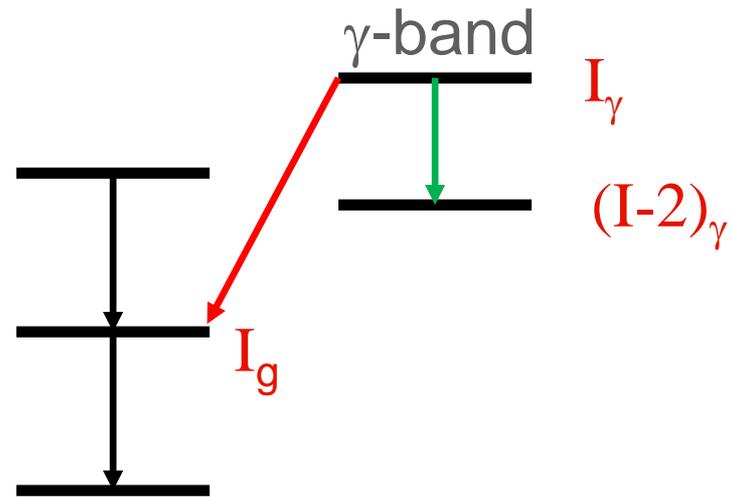
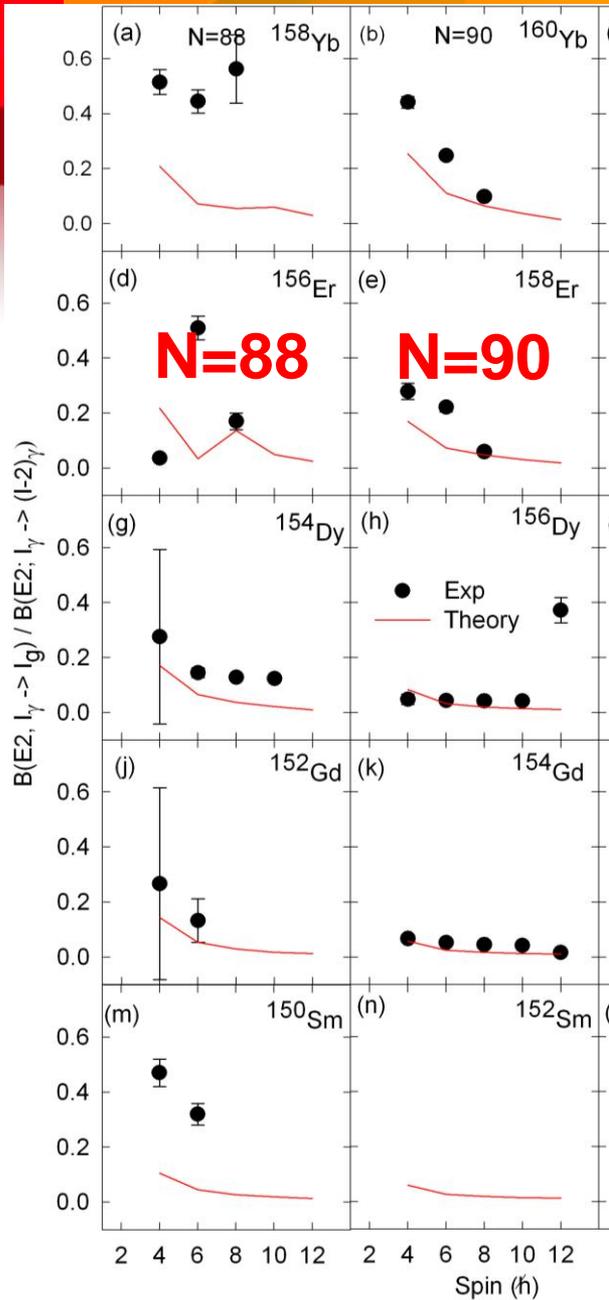
Branching Ratios β -band



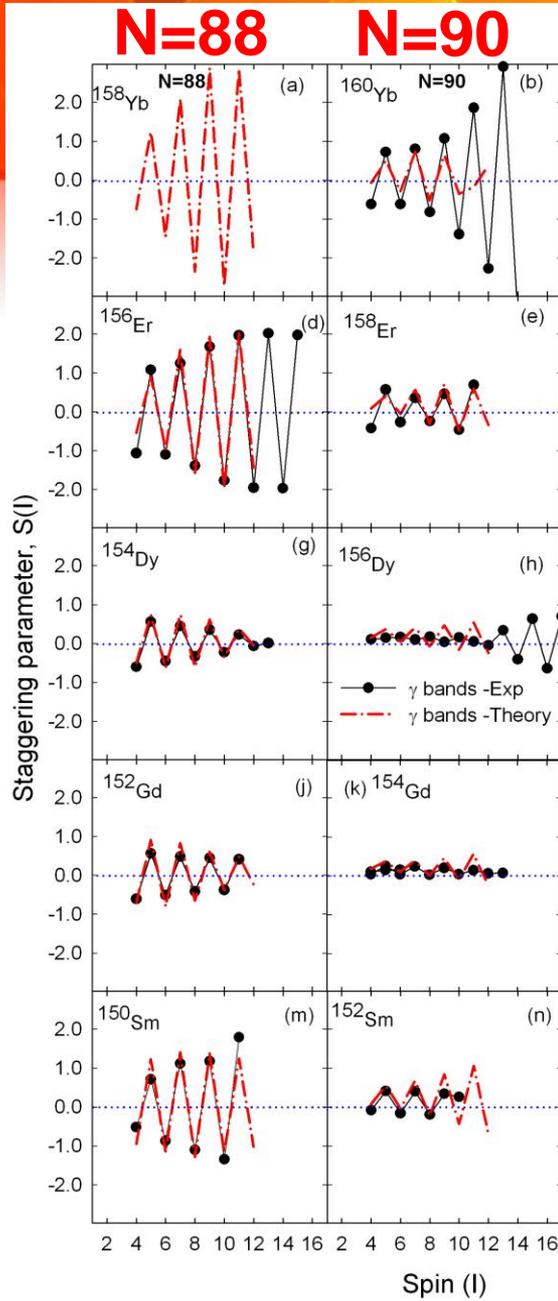
Branching Ratios γ -band



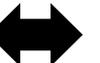
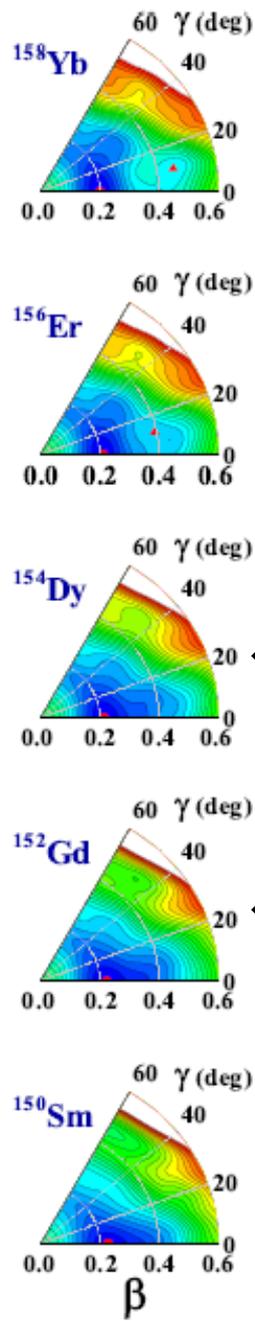
Branching Ratios γ -band



Staggering



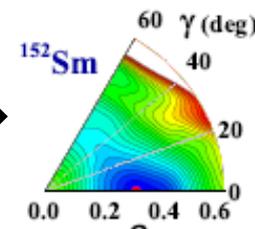
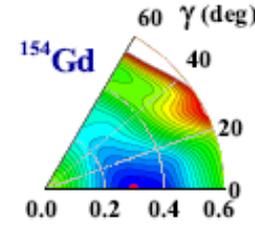
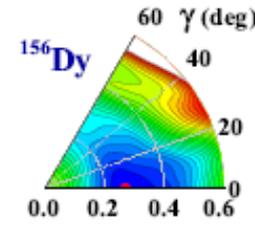
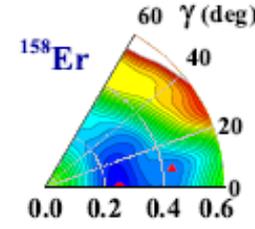
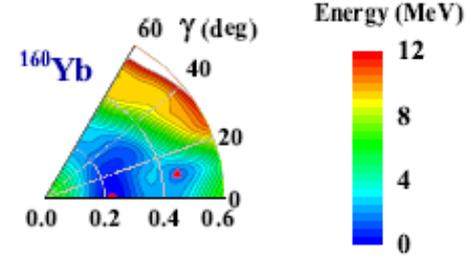
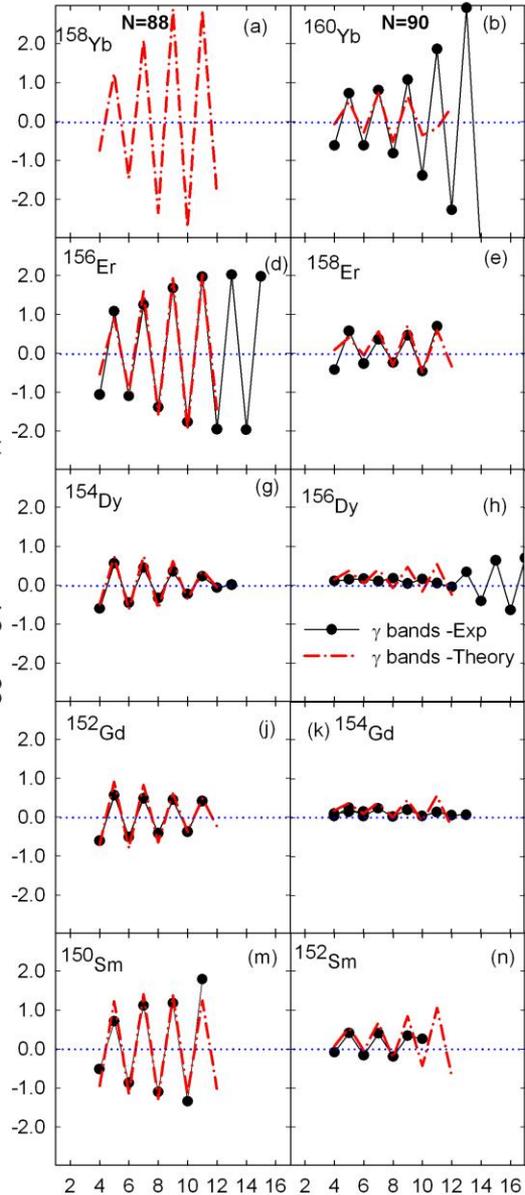
Staggering



N=88

N=90

Staggering parameter, $S(l)$

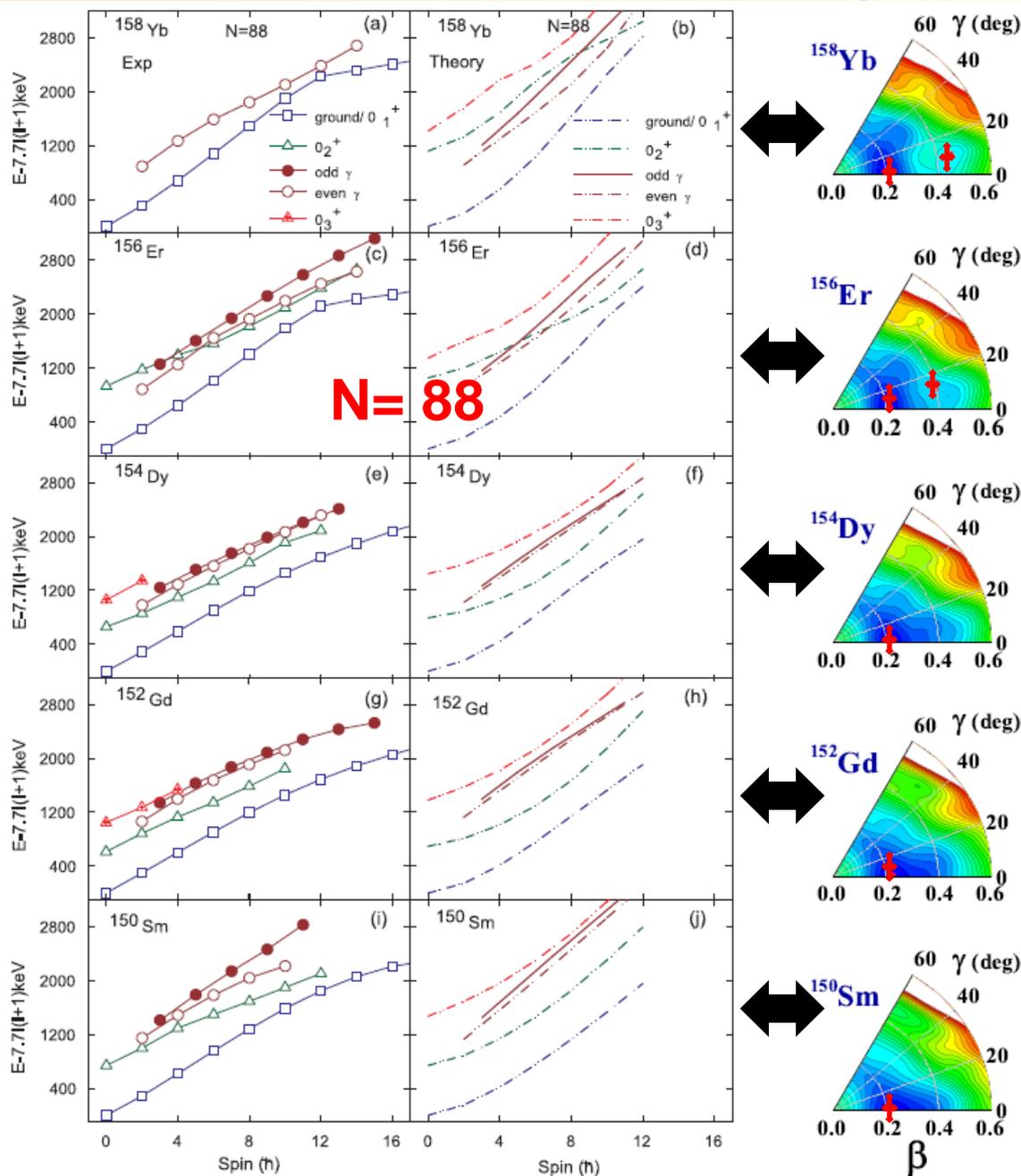


β

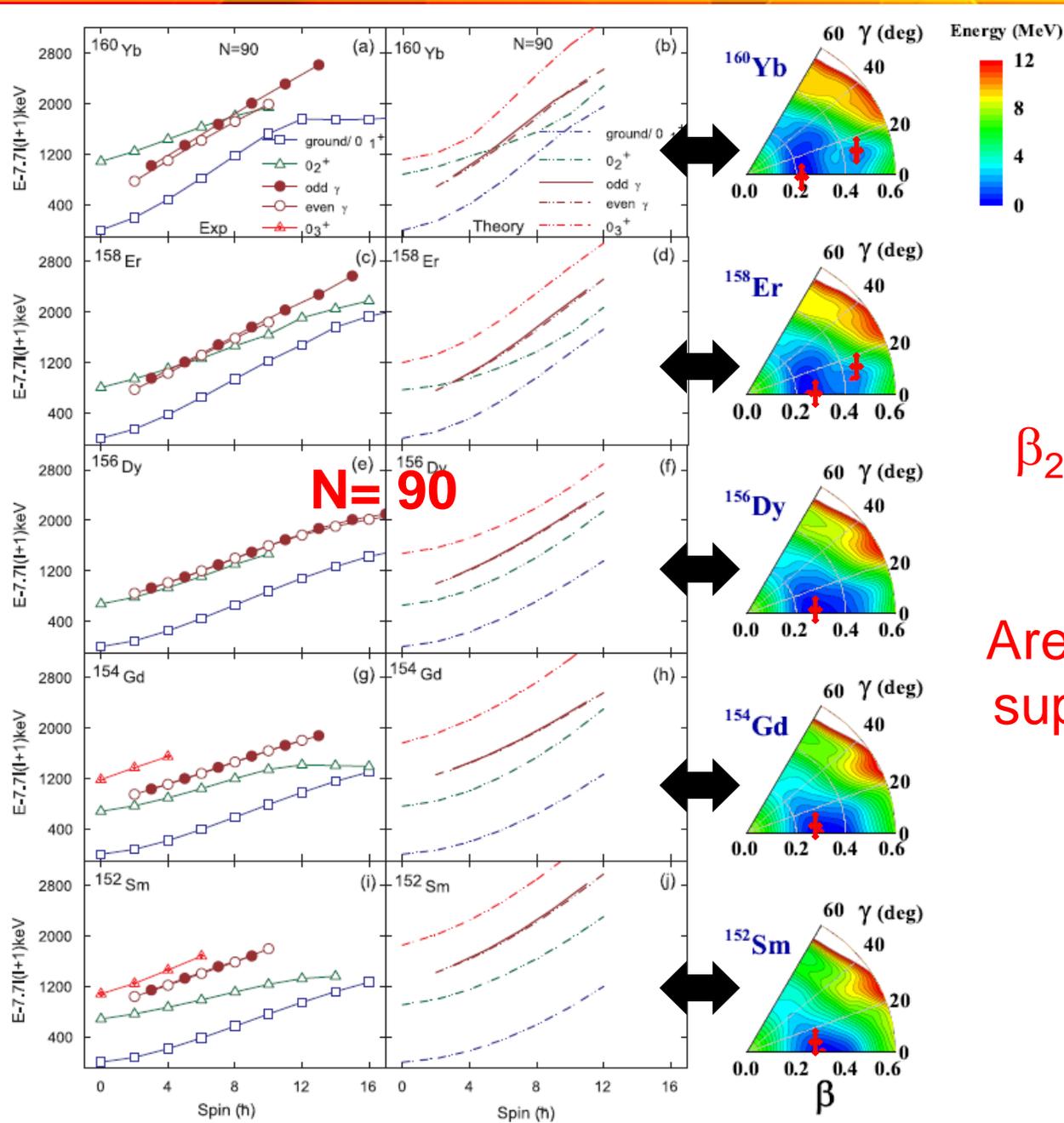
Spin (l)

β

Second minima in Er, Yb isotopes ($N=88$)



Second minima in Er, Yb isotopes (N=90)



$\beta_2 \sim 0.45$ and $\gamma \sim 10^\circ$ in Er, Yb isotopes

Are $|0_2^+\rangle$ states, Triaxially super deformed bands??

Conclusion

In order to produce a complete and definitive microscopic picture of the so-called β and γ bands, we have carried out an extensive systematic study of even-even nuclei in the 160 mass region.

Conclusion

Based on this data:

“From Sm to Dy, the $|0_2^+\rangle$ ” are well described as “ β -bands”

Maybe not “harmonic vibrations” but are essentially well described as solutions of the Bohr Hamiltonian with a realistic potential

Conclusion

Second minima for Er and Yb isotopes is located at
 $\sim\beta_2 \sim 0.45$ and $\gamma \sim 10^\circ$.

This may imply that the 0_2^+ bands in Er and Yb isotopes are Triaxially super deformed bands

Conclusion

Energy staggering and PES plots show that almost all the N=88 isotones from Sm to Yb are γ soft.

EXPERIMENTALISTS

Ntshangase

R. Bark

L. Mdletshe

A. Sithole

Bucher

T. Dinoko

E. Lawrie

J. Sharpey-Schafer

J. Lawrie

G. Zimba

+

AFRODITE CREW

JYVASKYLA

GAMMASPHERE...

iThemba LABS

University of Zululand

University of the Western Cape

University of Cape Town

Stellenbosch University

University of Johannesburg

RELATIVISTIC MEAN FIELDERS & COLLECTIVE MODELLERS:

Zhipan Li

Zhi Shi

Jie Meng

Bangyan Song

Chunyan Song

Jiangjing Yao

Shuangquan Zhang

.....

University of South West China

Peking University

Beihang University

Moment of Inertia: Inglis-Belyaev formula

$$\mathcal{I}_k = \sum_{i,j} \frac{(u_i v_j - v_i u_j)^2}{E_i + E_j} |\langle i | \hat{J}_k | j \rangle|^2 \quad k = 1, 2, 3,$$

Mass Parameters calculated in Cranking approximation

$$B_{\mu\nu}(q_0, q_2) = \frac{\hbar^2}{2} [\mathcal{M}_{(1)}^{-1} \mathcal{M}_{(3)} \mathcal{M}_{(1)}^{-1}]_{\mu\nu}$$

$$\mathcal{M}_{(n),\mu\nu}(q_0, q_2) = \sum_{i,j} \frac{\langle i | \hat{Q}_{2\mu} | j \rangle \langle j | \hat{Q}_{2\nu} | i \rangle}{(E_i + E_j)^n} (u_i v_j + v_i u_j)^2$$

Improved by using Thouless-Valatin moments-of-inertia

Z.P. Li. et al PRC 86, 034334 (2012)

Re-cast Bohr Hamiltonian

$$\hat{H} = \hat{T}_{\text{vib}} + \hat{T}_{\text{rot}} + V_{\text{coll}}$$

$$\hat{T}_{\text{vib}} = -\frac{\hbar^2}{2\sqrt{wr}} \left\{ \frac{1}{\beta^4} \left[\frac{\partial}{\partial \beta} \sqrt{\frac{r}{w}} \beta^4 B_{\gamma\gamma} \frac{\partial}{\partial \beta} - \frac{\partial}{\partial \beta} \sqrt{\frac{r}{w}} \beta^3 B_{\beta\gamma} \frac{\partial}{\partial \gamma} \right] + \frac{1}{\beta \sin 3\gamma} \left[-\frac{\partial}{\partial \gamma} \right. \right. \\ \left. \left. \times \sqrt{\frac{r}{w}} \sin 3\gamma B_{\beta\gamma} \frac{\partial}{\partial \beta} + \frac{1}{\beta} \frac{\partial}{\partial \gamma} \sqrt{\frac{r}{w}} \sin 3\gamma B_{\beta\beta} \frac{\partial}{\partial \gamma} \right] \right\}$$

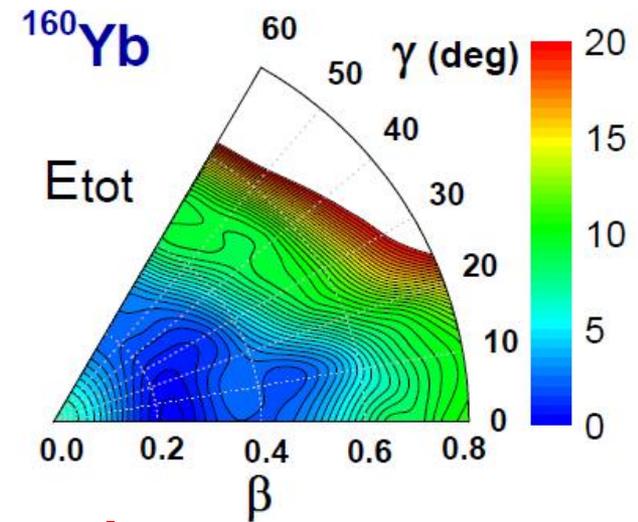
$$\hat{T}_{\text{rot}} = \frac{1}{2} \sum_{k=1}^3 \frac{\hat{J}_k^2}{\mathcal{I}_k}$$

Need to determine I 's, B 's

T. Niksic et al PRC 79, 034303 (2009)

Z.P. Li et al., PRC 79, 054301 (2009)

V_{coll} from Relativistic Mean Field



Parameters of PC-F1 functional
And pairing strengths adjusted to
ground observables of spherical
nuclei (binding energies, pairing
gaps, charge radii etc)

T. Niksic et al PRC 79, 034303 (2009)

Z.P. Li et al., PRC 79, 054301 (2009)