Singly closed shell nuclei:
Spherical nuclei and seniority-dominated coupling

I. Sketch of the shell model; basic signatures of seniority

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Reference: David J. Rowe and John L. Wood, Fundamentals of Nuclear Models, Foundational Models, Ch. 6 (World Scientific, 2010)—"R&W"

Some major goals of nuclear structure study

- We strive to understand nuclear structure:

  It is a quantum mechanical many-body problem

  It appears to be very complex

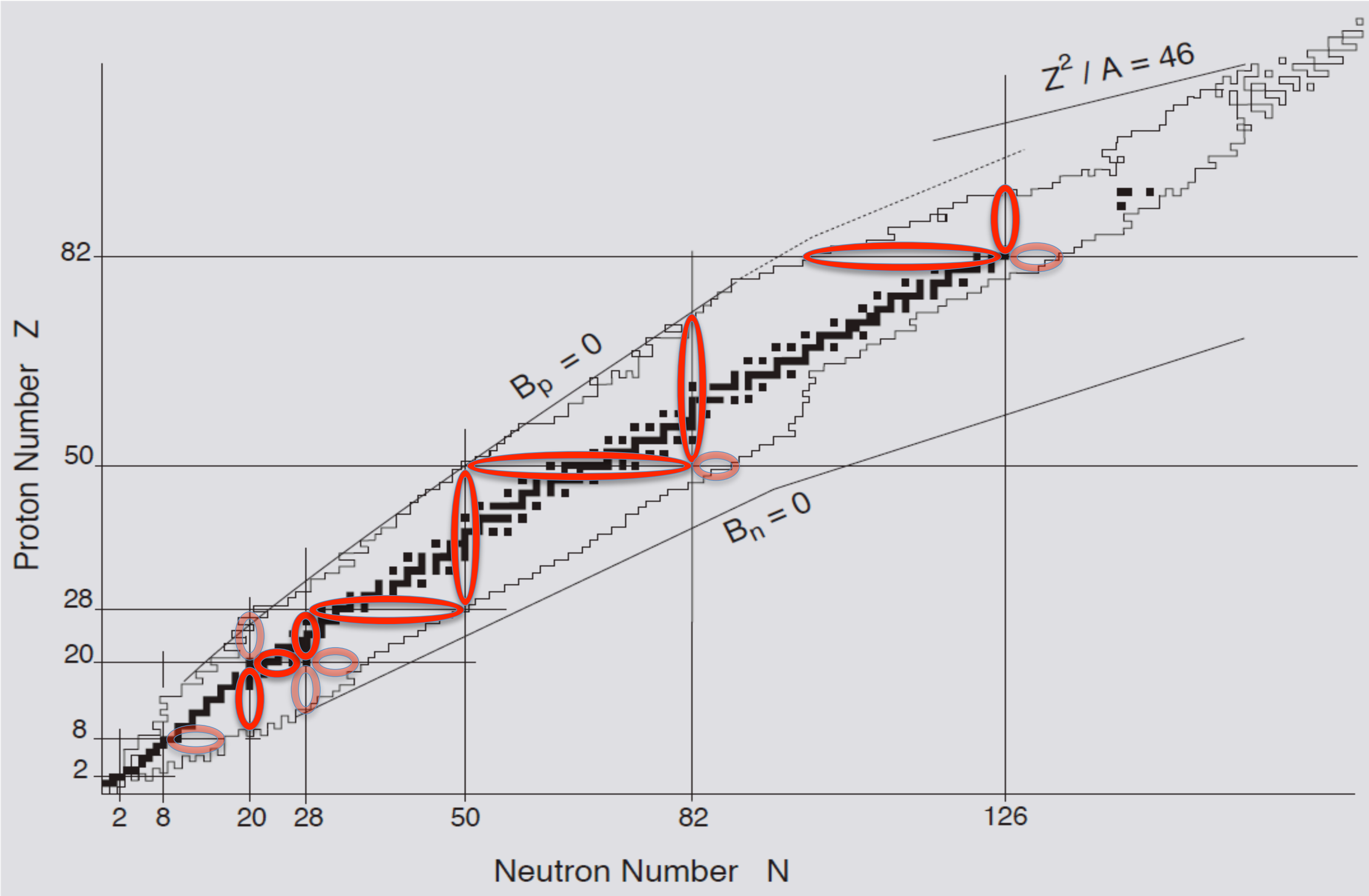
- We need to identify *simple patterns* of behavior:

  There is an extreme independent-particle view:
  the nuclear shell model

  We start by interpreting patterns in terms of
  independent particles and their interactions

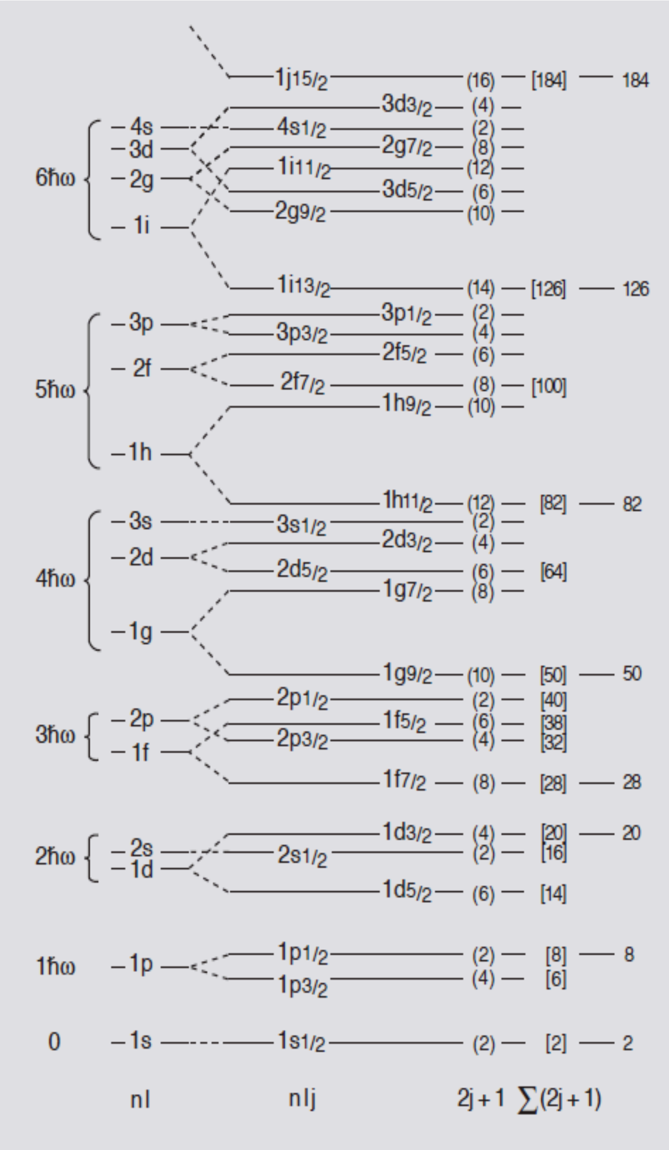
  We encounter correlations, e.g., “pairing”: this
  behavior is manifestly not “independent” particle.

Singly closed shell nuclei



R&W Fig. 1.2

Shell model: an *energy ordering*, and a *basis*



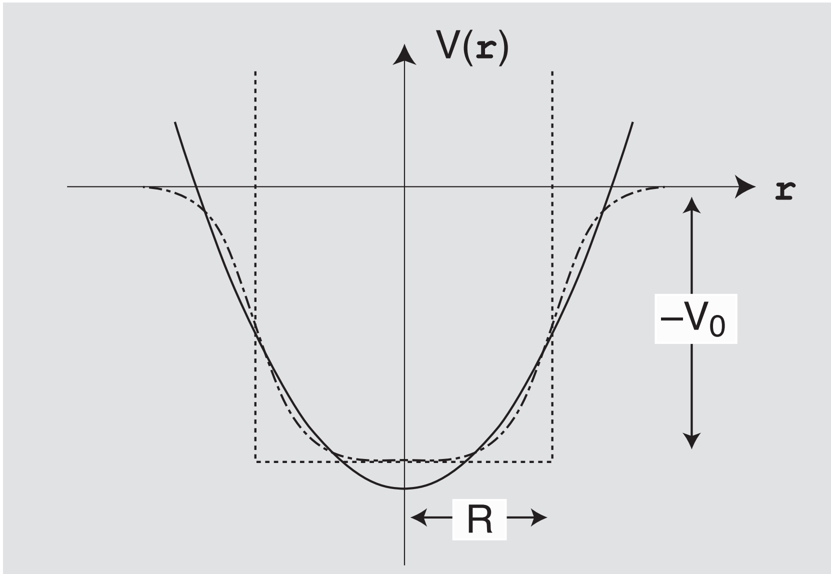
$$H = p^2/2m + V(r) \; {\color{red} + \, C\, l \bullet s}$$

$$C < 0$$

Harmonic oscillator, $V(r) = \frac{1}{2}\, m\, \omega^2 r^2$

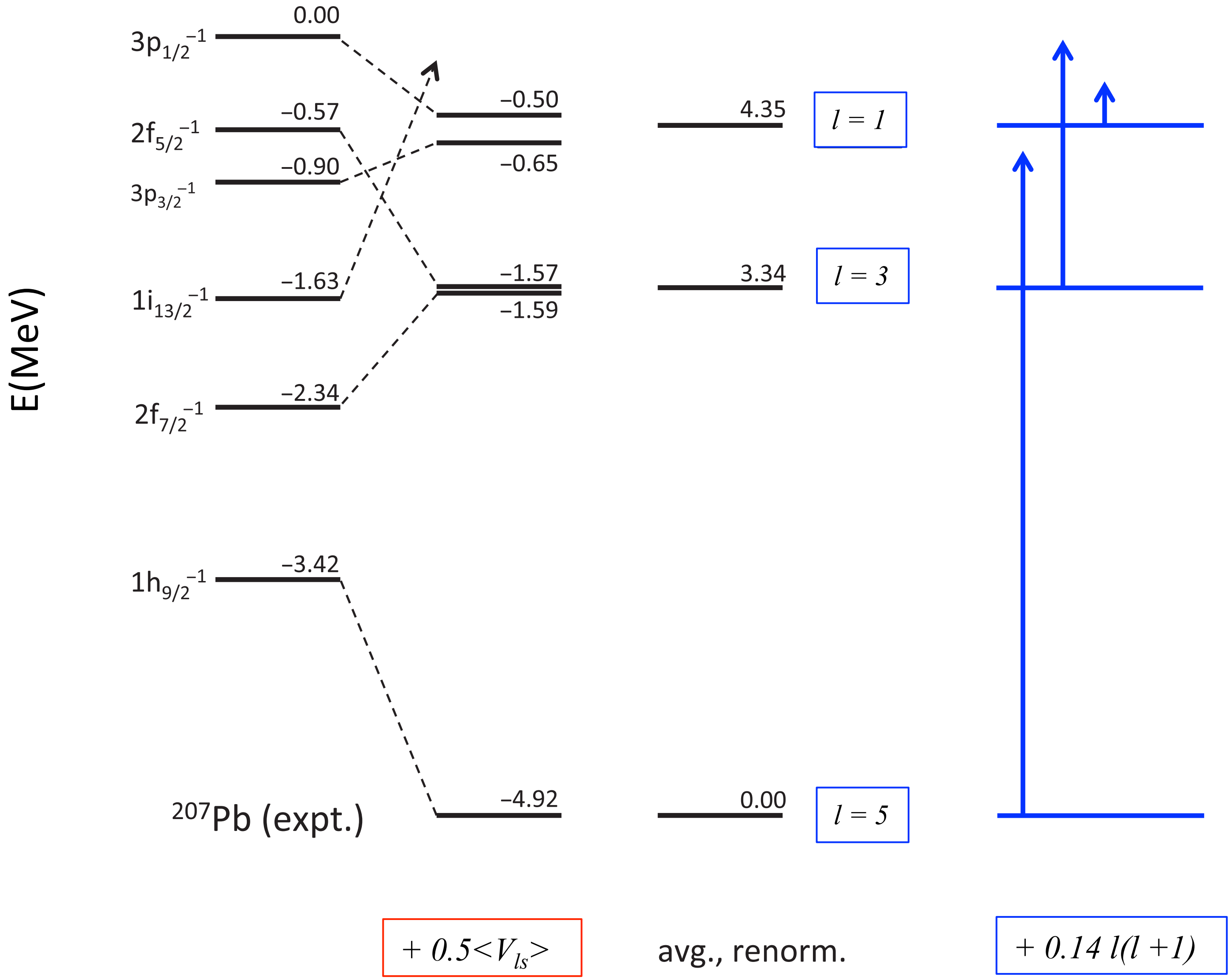
Woods-Saxon, $V(r) \sim -\; \rho(r)$

Infinite square well, $V(r) = const.,\; r < R$



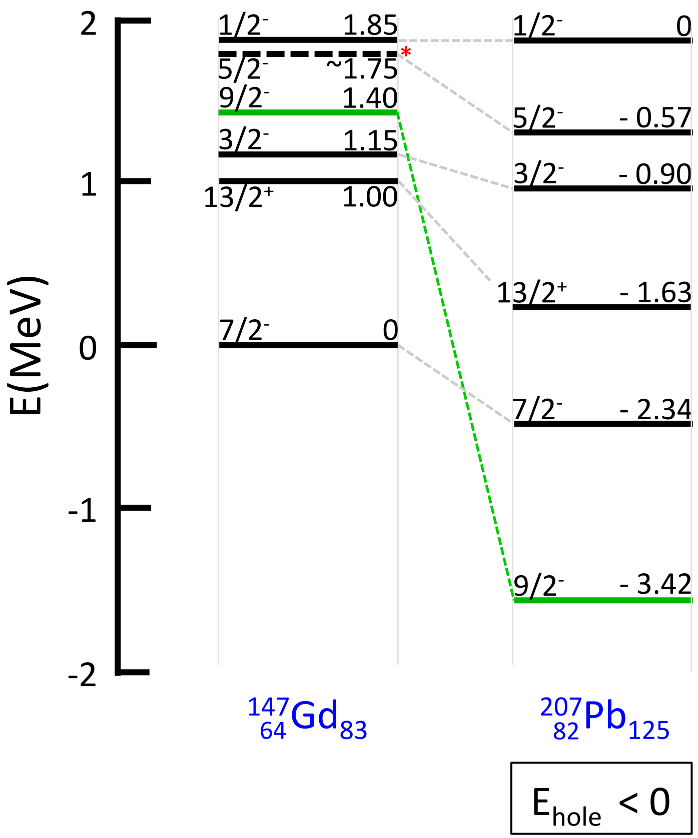
R&W Figs. 1.14, 1.17

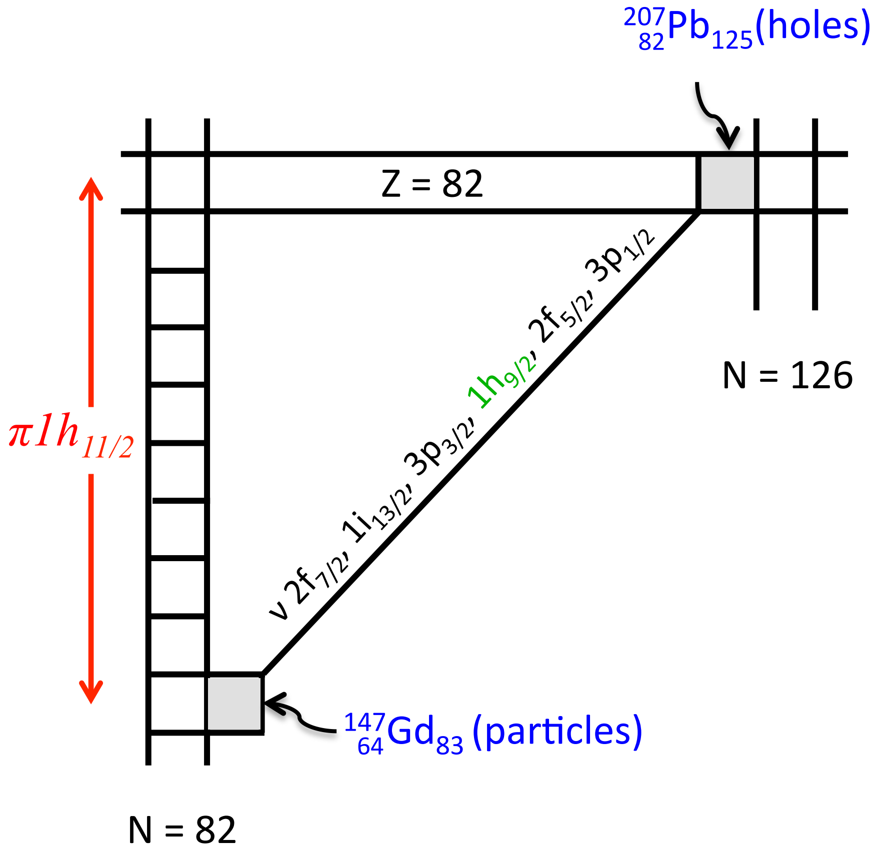
Removal of spin-orbit splitting to reveal non-degeneracy of subshell energies and *l* dependence



Independent-particle potentials are only part of the physics, even adjacent to closed shells

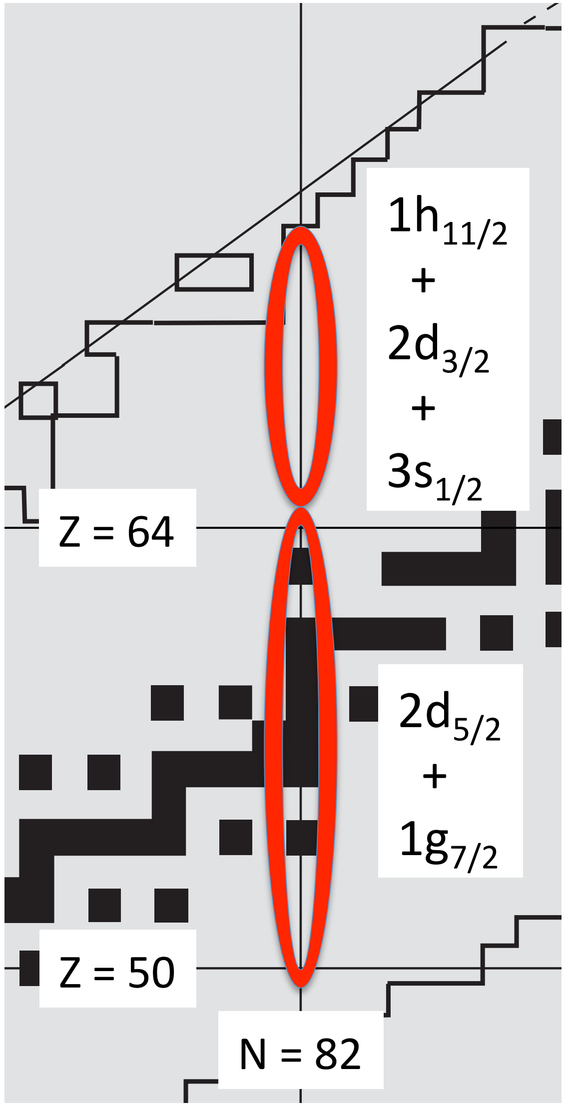
$$<\nu\, 1h_{9/2}|V_{\pi\nu}^{monopole}|\pi 1h_{11/2}>$$

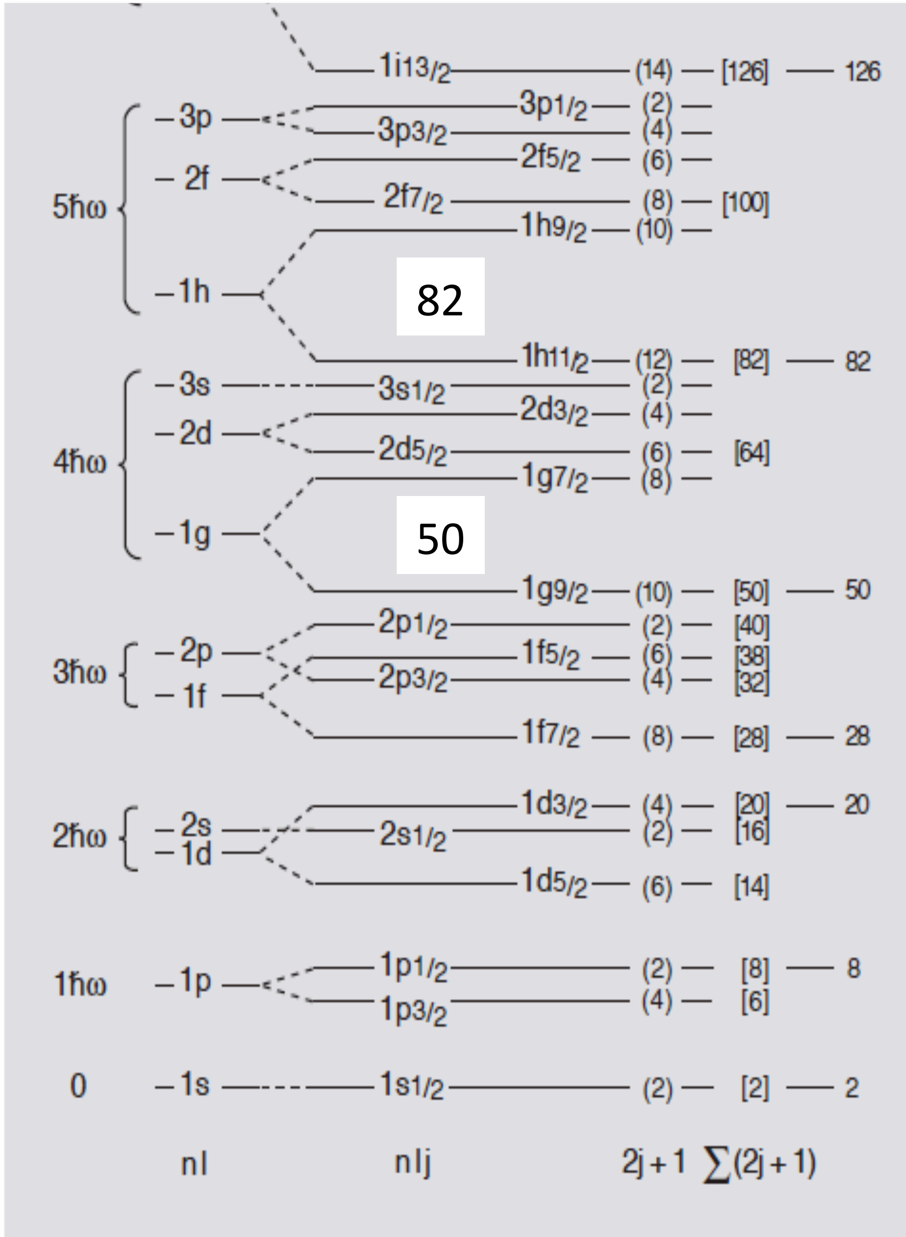




*estimate from N = 83 systematics

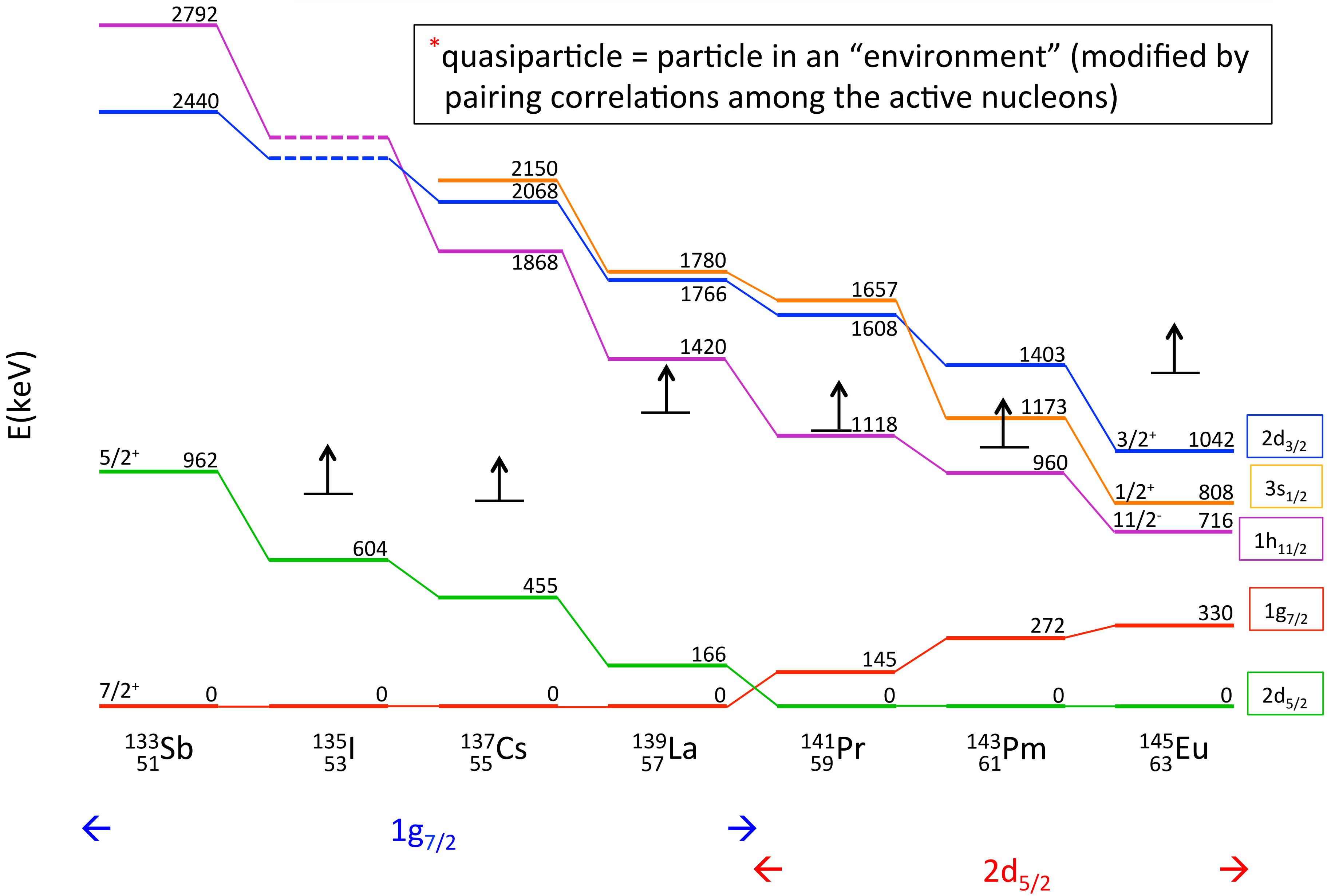
N = 82 singly closed shell nuclei: shell model



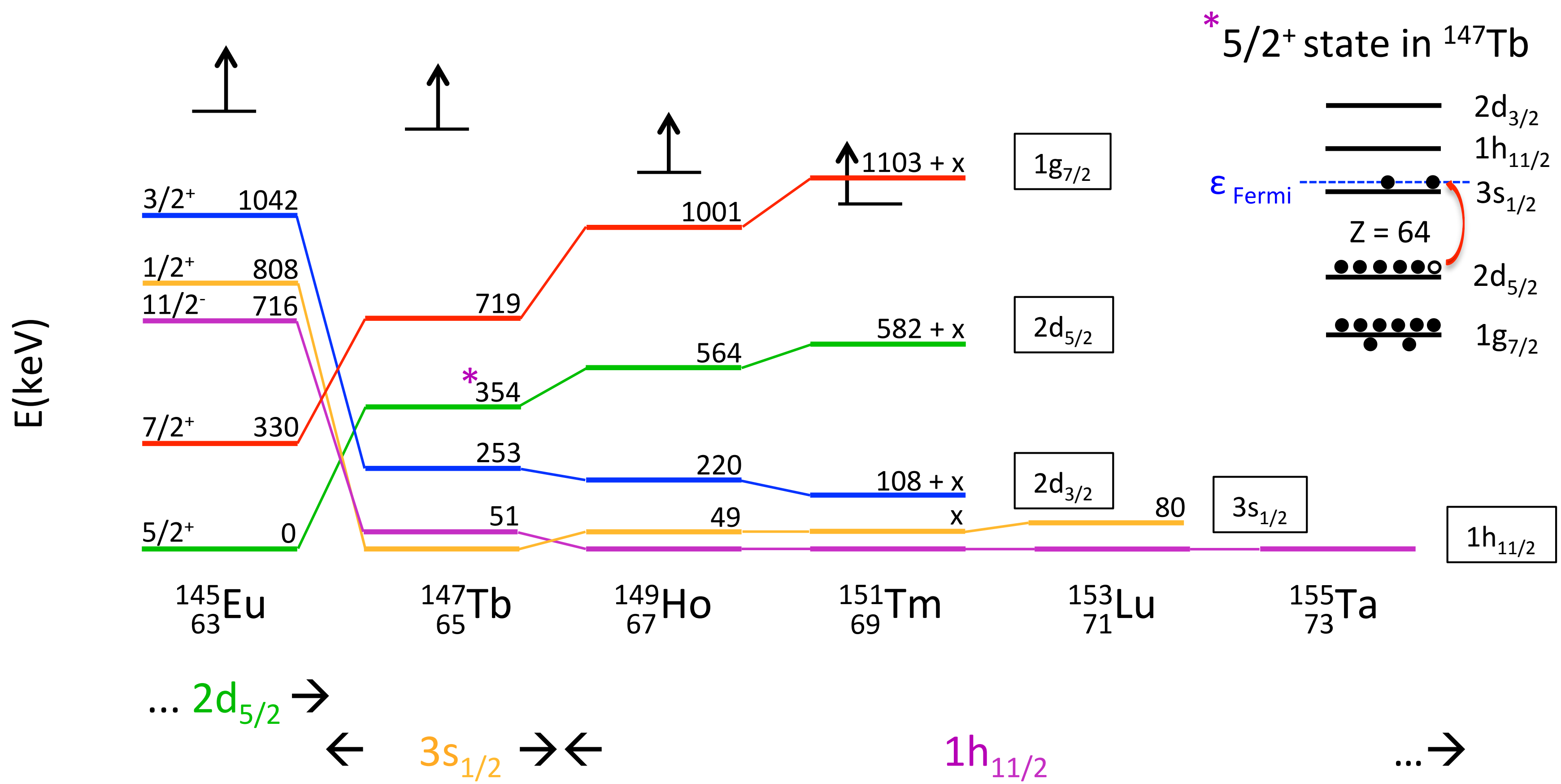


N = 82: proton single-quasiparticle* states, Z < 64

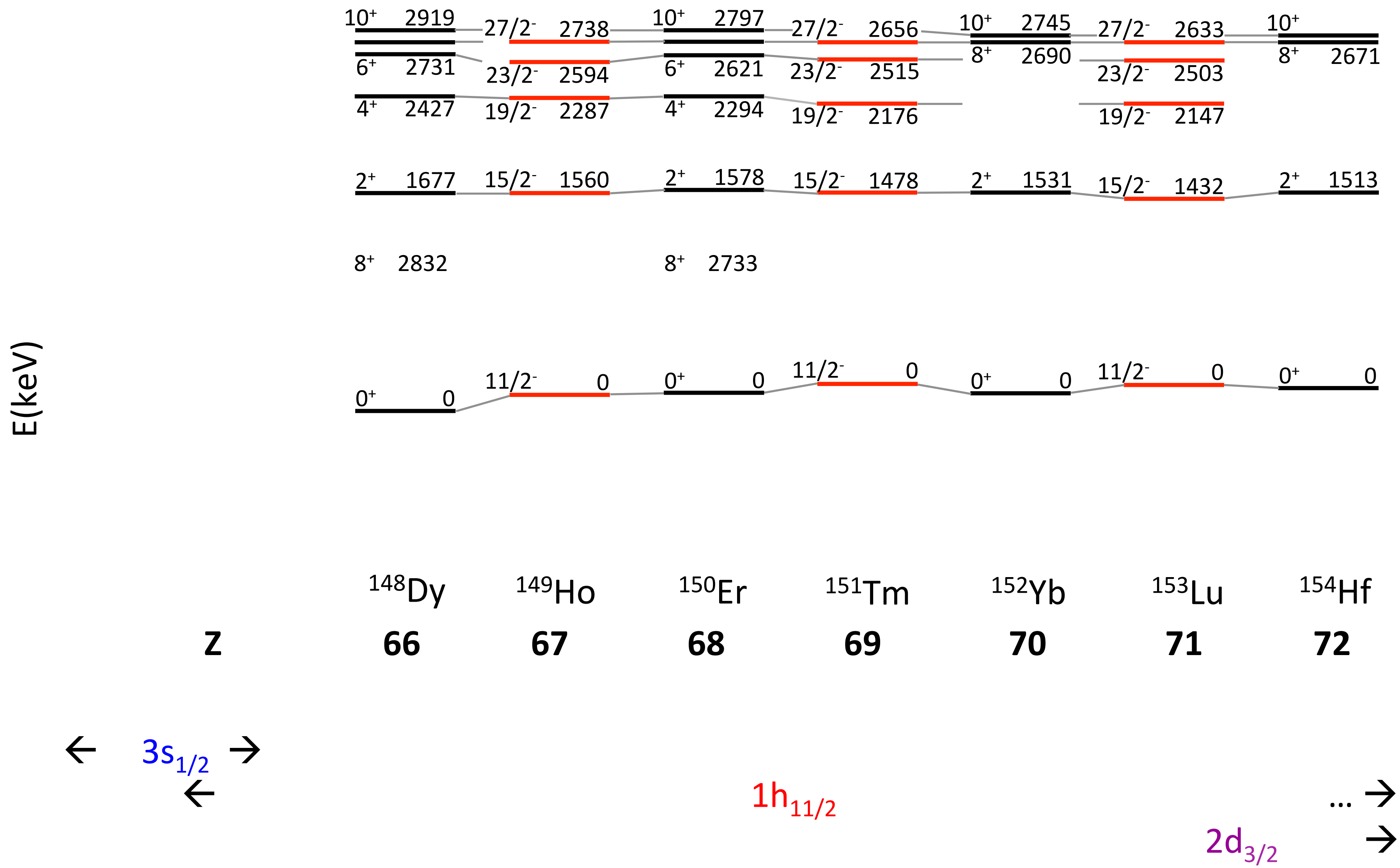
*quasiparticle = particle in an “environment” (modified by pairing correlations among the active nucleons)



N = 82: proton single-quasiparticle states, Z > 62



N = 82: proton “seniority” structure, Z > 65



Simple, persistent structures in complex systems demand simple explanations!

Handling many-fermion configurations:
the "m-scheme" and allowed total spin couplings for j^2

Pauli principle:

1). two identical fermions cannot have all quantum numbers the same

2). fermionic states must be anti-symmetric, viz. $|m_1 m_2> - |m_2 m_1>$

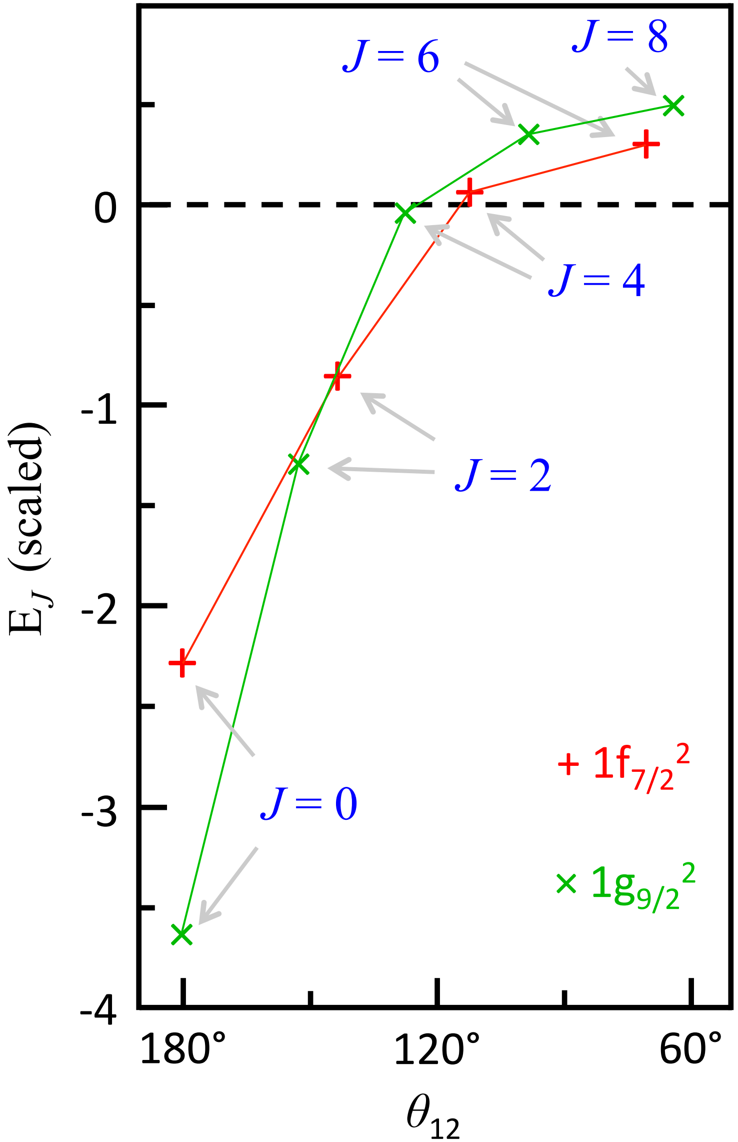
Ex. j_1=3/2, j_2=3/2

No J = 1, 3

J=2, 0

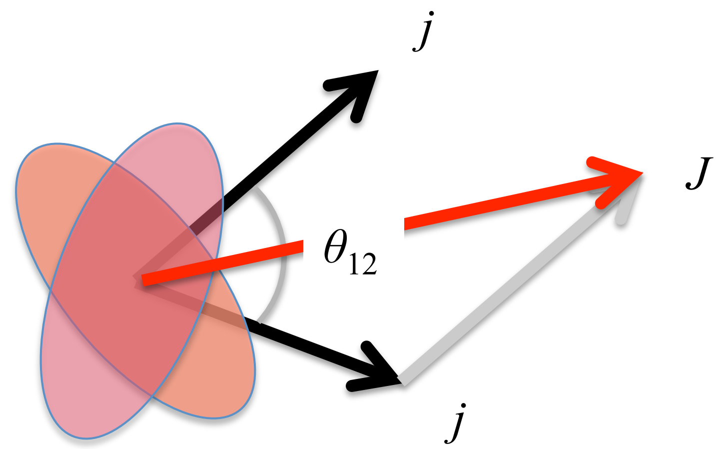
m_1	m_2	M
+3/2	+1/2	+2
+3/2	-1/2	+1
+3/2	-3/2	0
+1/2	-1/2	0
+1/2	-3/2	-1
-1/2	-3/2	-2

j^2 pair coupled to resultant J, diagonal energies:
short-range force, *overlap* of wave functions



$j_1 = j_2 = j$, J = EVEN, identical nucleon pair

$$\cos\theta_{12} = \frac{J(J+1)}{2j(j+1)} - 1$$

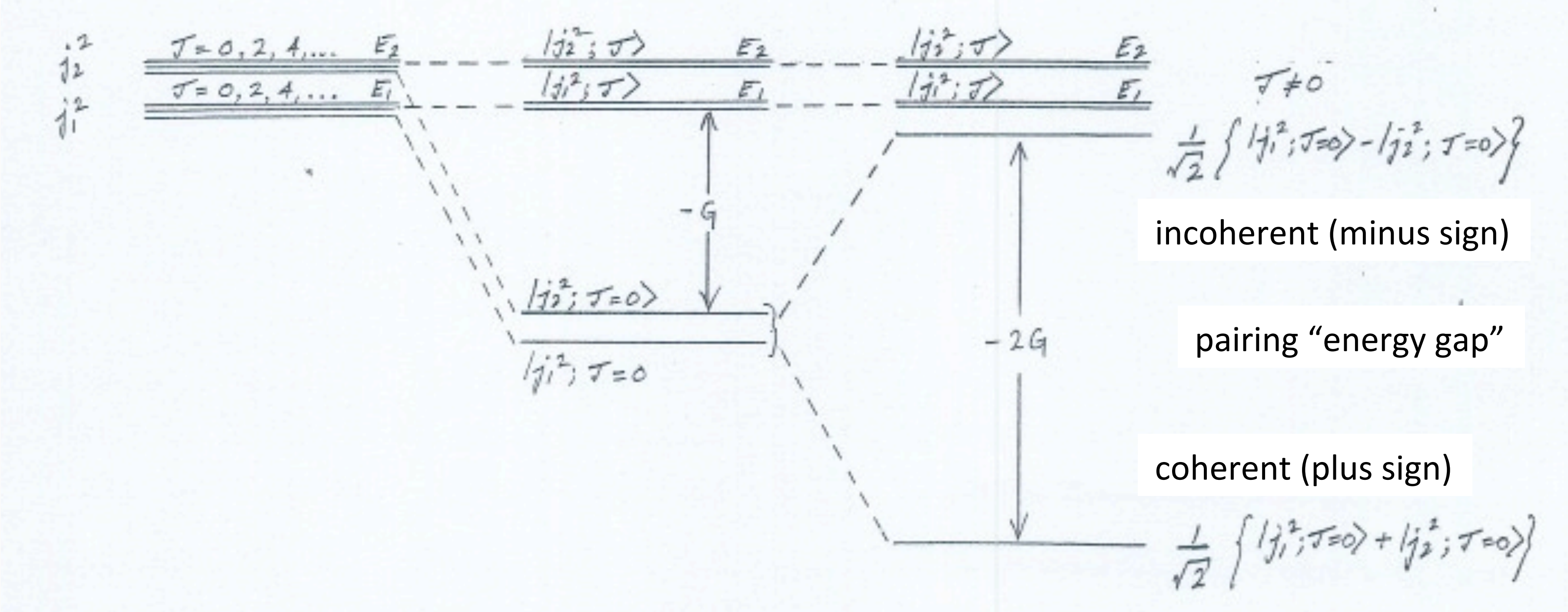


semi-classical view of wave function overlap
NOTE: anti-symmetrization weakens high-J interactions

J.P. Schiffer and W.W. True, Rev Mod Phys **48** 191 (1976);
W.W. Daehnick, Phys Repts **96** 317 (1983)

Multi-j J = 0 pair states:
pairing—diagonal and off-diagonal energies

Simplest pairing force model: one pair

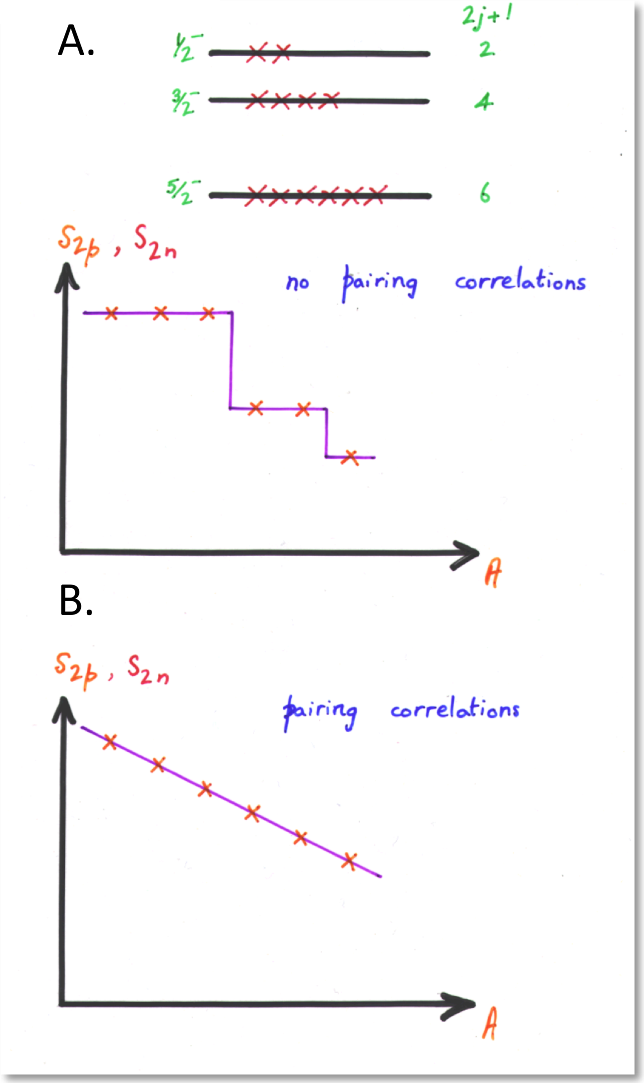


$$H_0 = \begin{pmatrix} E_1 & 0 \\ 0 & E_2 \end{pmatrix} \qquad \begin{pmatrix} E_1 & 0 \\ 0 & E_2 \end{pmatrix} \qquad \begin{pmatrix} E_1 & 0 \\ 0 & E_2 \end{pmatrix} \qquad E_1 \approx E_2 \equiv 0$$

$$+$$

$$V \quad \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \qquad -G_{J=0}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \qquad -G_{J=0}\begin{pmatrix} 1 & \boxed{1} \\ \boxed{1} & 1 \end{pmatrix} \qquad G_{J=0} \equiv G$$

The anatomy of pairing in finite many-body quantum systems: effect on separation energies



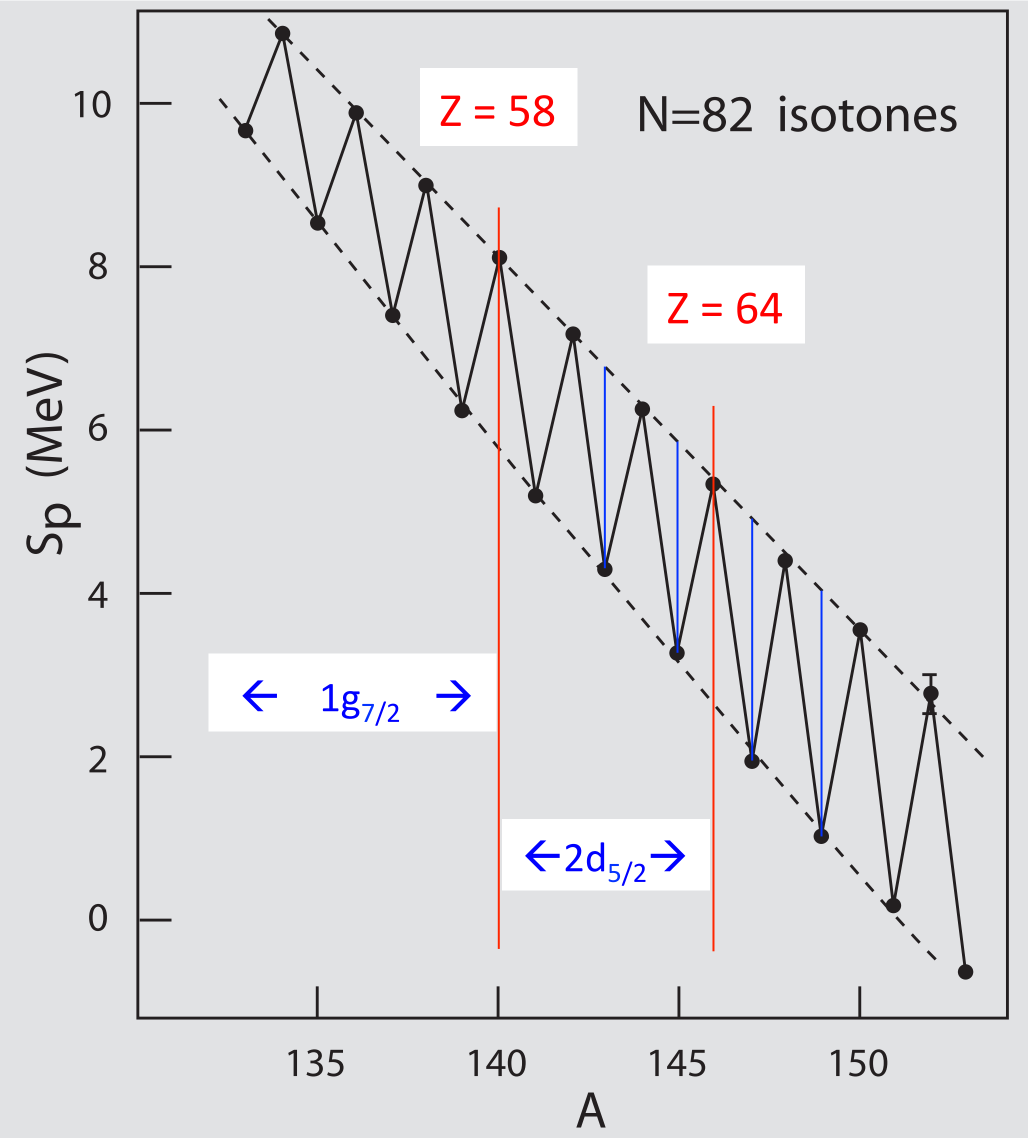
A. Two-proton separation energies
in a shell model picture

B. Two-proton separation energies
in a shell model + pairing model picture

$|gs; n\ pairs\rangle = \{|j_1^2, J=0\rangle + |j_2^2, J=0\rangle + \ldots\}^n$

--"Cooper" pairs

One-proton separation energies, S_p: N = 82



Pairing correlations:
odd-even staggering;
no discontinuities at subshells

|odd gs> = |j_i> x |gs; n pairs>

j_i "blocks" state i
--not accessible to J = 0 pairs,
reduces correlation energy

R&W Fig. 6.21