Singly closed shell nuclei:
Spherical nuclei and seniority-dominated coupling
II. Multi-j shell seniority
Singly closed shell nuclei

Case study: $N = 82$ multiple $j$ subshells
N = 82 singly closed shell nuclei: shell model

Z = 64
1h_{11/2} + 2d_{3/2} + 3s_{1/2}

Z = 50
2d_{5/2} + 1g_{7/2}

N = 82

N = 82

1 Elements of nuclear structure

Figure 1.2: Chart of the nuclides (ca. 2005). The black squares denote the nuclei found in nature. The present extent of nuclei, for which at least one characteristic has been measured, are enclosed by the "stepped" border. The line marked \( B_n = 0 \) approximate the neutron drip line. The line marked \( B_p = 0 \) approximate the proton drip line. Outside of these borders, neutrons and protons are lost so rapidly from nuclei that the nuclei cannot be observed. The line marked \( Z^2/A = 46 \) indicates a approximate limitation on the nucleus stability with respect to spontaneous fission. The magic numbers 2, 8, 20, 28, 50, 82, and 126 (cf. Section 1.3) also shown.

Before they were observed. There have been many experimental surprises. In this respect, nuclear physics is in "good company" with condensed matter physics. There are a number of parallels. For example, many nuclei and condensed matter systems exhibit superfluidity. In fact, it is sensible to regard nuclei as examples of condensed matter.

The future of nuclear physics, as also of condensed matter physics, depends in part on producing new condensates. For nuclei this means new combinations of N and Z. A consideration of Figure 1.2 reveals that we have studied only about 30% of the possibilities. To study the remaining 70% of the possibilities, which are very unstable, is one of the future challenges of nuclear physics.
N = 82: proton single-quasiparticle* states, Z < 64

*quasiparticle = particle in an “environment” modified by pairing correlations among the active nucleons
$N = 82$: proton $1g_{7/2}$ and $2d_{5/2}$ seniority*, $\nu = 1$ structure

*seniority, $\nu$ = number of unpaired particles
**N = 82: proton $1g_{7/2}$ seniority**, $\nu = 1, 3$ structure

*from m scheme; $M = +7/2 + 5/2 + 3/2 = 15/2$

Note: spin of 1010 keV state in $^{135}$I is not known, but is consistent with $3/2^+$. A state with spin-parity $3/2^+$ is predicted at about 1 MeV excitation energy in $^{137}$Cs.

Pauli blocking forbids other $J$ values.

Note: no $J = 7/2$, contrary to $j = 7/2 \times 2^+$; thus, $2^+$ is a simple broken-pair state.

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*seniority, $\nu =$ number of unpaired particles*
$N = 82$: proton $1g_{7/2}$ seniority, $\nu = 0, 2$ structure

NOTE: excitation patterns are not strongly dependent on particle number, only on Pauli principle.
N = 82: proton 1g_{7/2} and 1g_{7/2} + 2d_{5/2} seniority, v = 0, 2 structure


1g_{7/2}^{n-1} 2d_{5/2}^{1} v = 2, J^* = 1, ...6

NOTE: excitation patterns are not strongly dependent on particle number, only on Pauli principle

*from m scheme; but classical vector coupling can be used for j_1 \neq j_2:

\[ J = \frac{7}{2} - \frac{5}{2} = 1 \]

\[ \ldots \]

\[ J = \frac{7}{2} + \frac{5}{2} = 6 \]

\begin{center}
\begin{tabular}{lcccc}
\hline
 & 0^+ & 1^+ & 2^+ & 4^+ \\
0^+ 1g_{7/2} & 0 & 0 & 0 & 0 \\
0^+ 1g_{7/2} & 134\textsuperscript{Te} & 136\textsuperscript{Xe} & 138\textsuperscript{Ba} & 140\textsuperscript{Ce} \\
Z & 52 & 54 & 56 & 58 \\
n & 2 & 4 & 6 & 8 \\
\hline
\end{tabular}
\end{center}
$1g_{7/2} - 2d_{5/2} \nu = 2$ multiplet in the $N = 82$ isotones

$g_{7/2}^4, \nu = 4$: $J = 2, 4, 5, 8$

NEW PERSPECTIVE

NEW RESULT: Peters
$N = 82$: proton $1g_{7/2}$ seniority, $\nu = 0, 2, 4$ structure


$1g_{7/2}^4 \ \nu = 4, J = 2, 4, 5, 8^*$

$* from m scheme$

$v = 4$ not at twice the excitation of $v = 2$ because of Pauli blocking.

$J^+ \ 1g_{7/2}$
$v = 2$

$6^+$
$4^+$
$2^+$

$1g_{7/2}^2$
$v = 0$

$0^+$

$134\text{Te}$ $136\text{Xe}$ $138\text{Ba}$ $140\text{Ce}$

$Z$ $52$ $54$ $56$ $58$

$1g_{7/2}^2 \ 1g_{7/2}^4 \ 1g_{7/2}^6 \ 1g_{7/2}^6 \ 2d_{5/2}^2$

$1g_{7/2}^6 \approx 1g_{7/2}^{-2}$
N = 82: proton seniority* structure, Z < 63, $1g_{7/2}2d_{5/2}$


*seniority, $v = \text{number of unpaired particles}$

*Comprehensive (complete) spectroscopy up to spin 6
N = 82 excited $0^+_2$ states and $B(E2; 2^+_1 \rightarrow 0^+_1)$ strengths

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<th>Z</th>
<th>$B_{20}$(W.u.)</th>
<th>$^{134}$Te</th>
<th>$^{136}$Xe</th>
<th>$^{138}$Ba</th>
<th>$^{140}$Ce</th>
<th>$^{142}$Nd</th>
<th>$^{144}$Sm</th>
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<td>16.6$^{24}$</td>
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$B_{42}$ vs. $B_{20}$ for singly closed-shell nuclei

$N = 82$

$B_{20} := B(E2; 2_1^+ \rightarrow 0_1^+)$

$B_{42} := B(E2; 4_1^+ \rightarrow 2_1^+)$
Vibrations: a schematic view

Transition probabilities between collective vibrational states should be proportional to the number of phonons.

\[ a \dagger |0\rangle = |1\rangle \]

\[ a \dagger |n\rangle = \sqrt{n+1} |n+1\rangle \]

\[ T(n \rightarrow n+1) = b \langle n+1|a \dagger |n\rangle^2 = b(n+1) \]
B_{64} vs. B_{20} for singly closed shell nuclei

No multi-phonon quadrupole collective strength

$$B_{64} := B(E2; 6_1^+ \rightarrow 4_1^+)$$
CONCLUSIONS

• N = 82, singly closed shell nuclei with Z < 65 exhibit only seniority-dominated excitations at low energy
• This is for two j shells (1g_{7/2}, 2d_{5/2})—a “first”
• A comprehensive (complete) view up to ~ 2500 keV has been achieved
• No multi-phonon vibrational degrees of freedom are present

• This perspective suggests that simple, seniority-dominated multi-j excitations be explored in all singly closed shell nuclei