Rotation-Particle Coupling

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Fundamentals of Nuclear Models: Foundational Models
--David J. Rowe and JLW, World Scientific, 2010 [R&W]
+ second volume nearing completion

Rotation-Particle Coupling

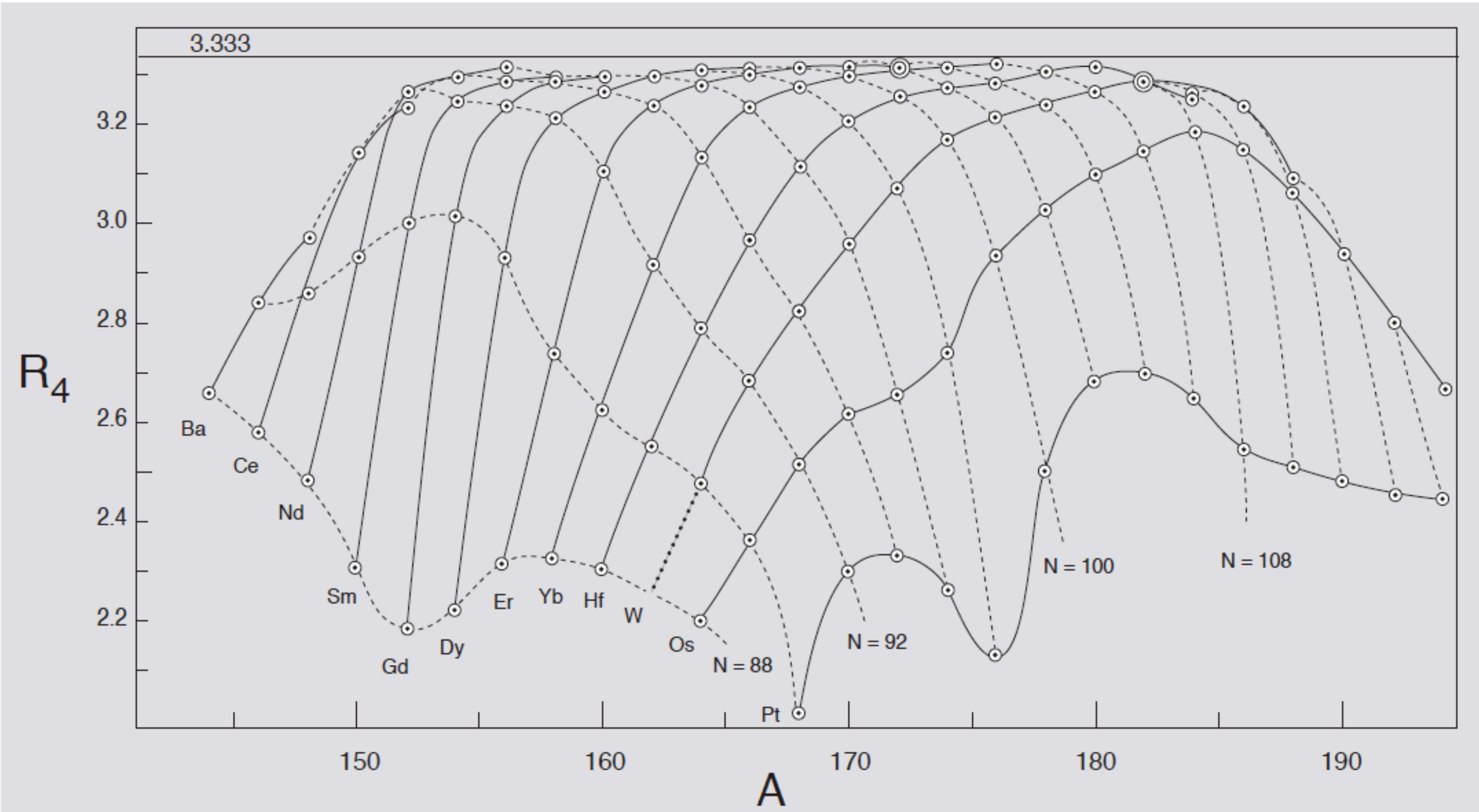
- Many (most) nuclei are deformed and rotate
- Some nuclei with spherical ground states possess deformed excited states
- All odd and odd-odd nuclei possess unpaired particles
- Above the "pairing" energy gap, all even-even nuclei possess unpaired particles

UNDERSTANDING ROTATIONAL-PARTICLE COUPLING IS
A FUNDAMENTAL STEP IN THE STUDY OF NUCLEAR STRUCTURE

Rigid rotor model energy ratios

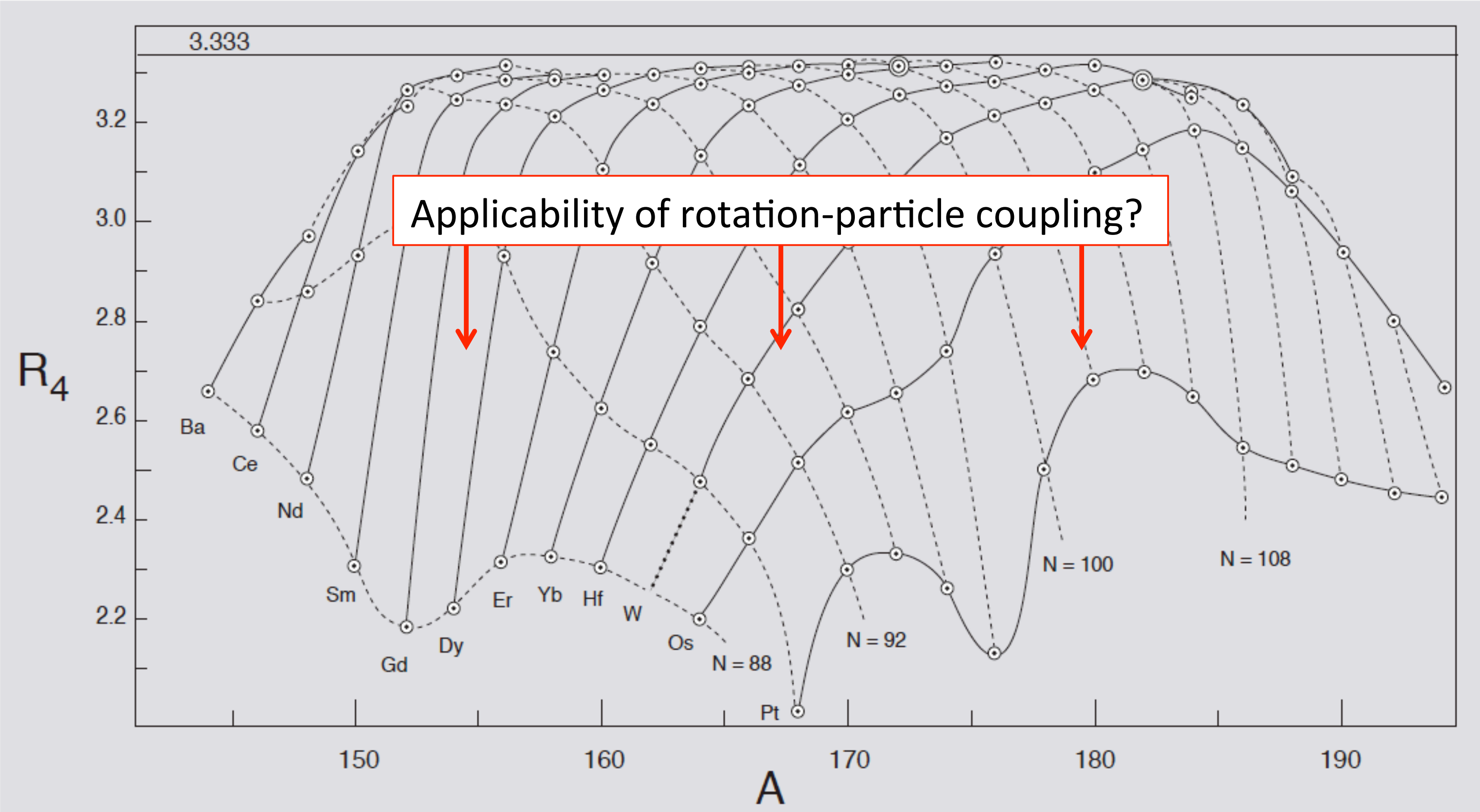
$R_4 = E(4_1^+) / E(2_1^+)$ $E(I) = A\,I(I + 1)$

$A \sim (\text{moment of inertia})^{-1}$



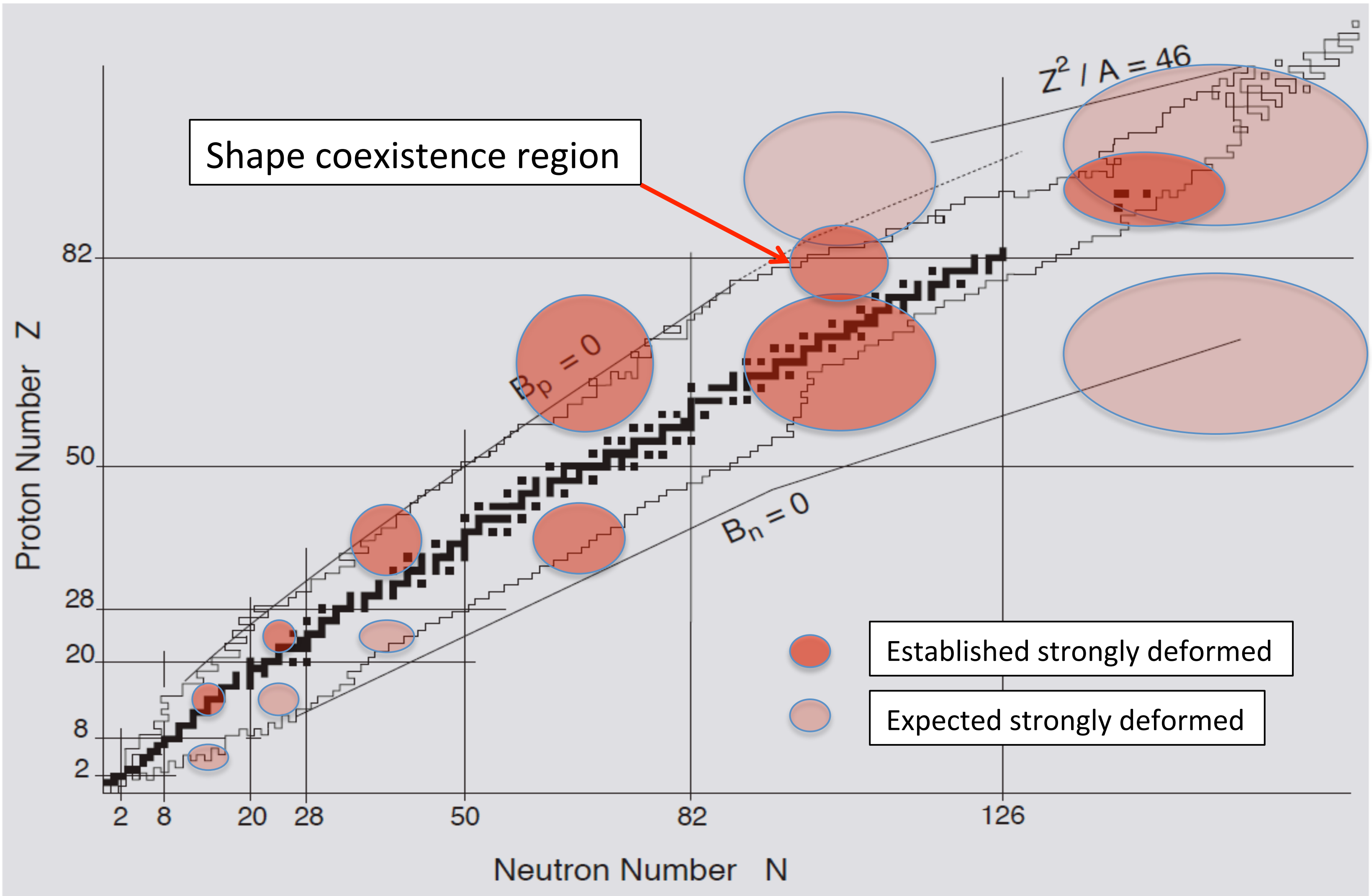
R&W Fig. 1.56

What should be called a rotor?



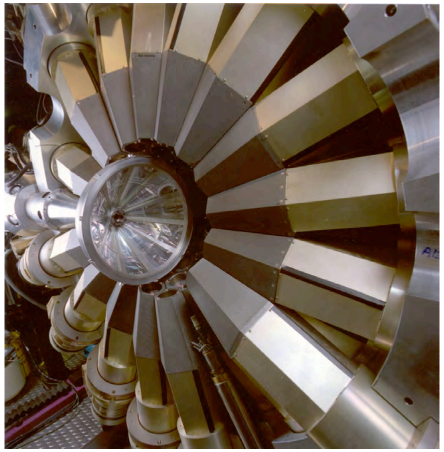
R&W Fig. 1.56

Strongly deformed nuclei



Multi-step Coulomb Excitation

Multiple-step Coulomb excitation (multi-Coulex) populates excited collective states in nuclei.



Gammasphere:
110 Ge γ-ray detectors.

CHICO:
particle detector.

An incident nucleus is scattered by the Coulomb interaction with a target nucleus.

A close ("safe") approach results in multiple Coulomb interactions only (no complication from strong interactions).

Level energies and transition strengths are deduced through γ-ray spectroscopy.

Level lifetimes, quadrupole moments and transition matrix elements are deduced by a least-squares fit to γ-ray yields.

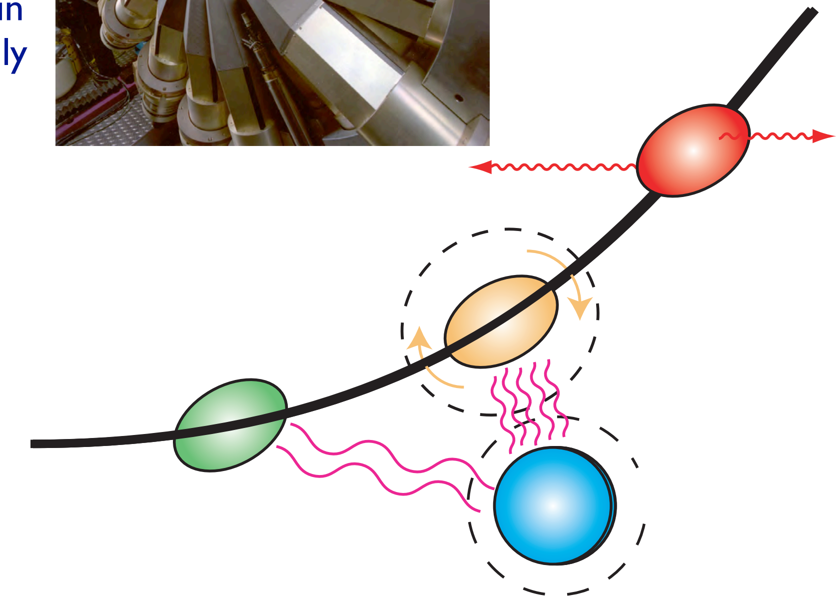
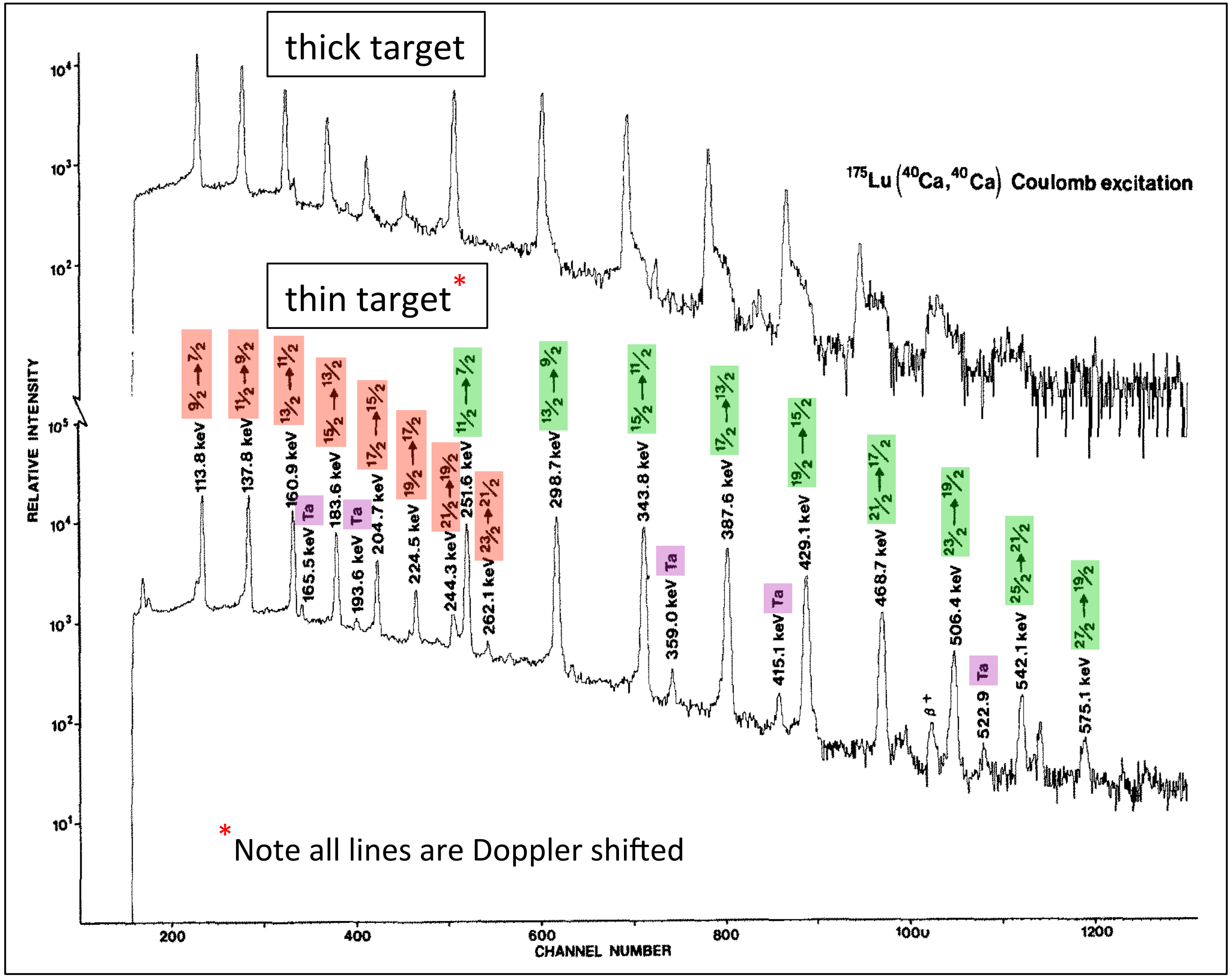
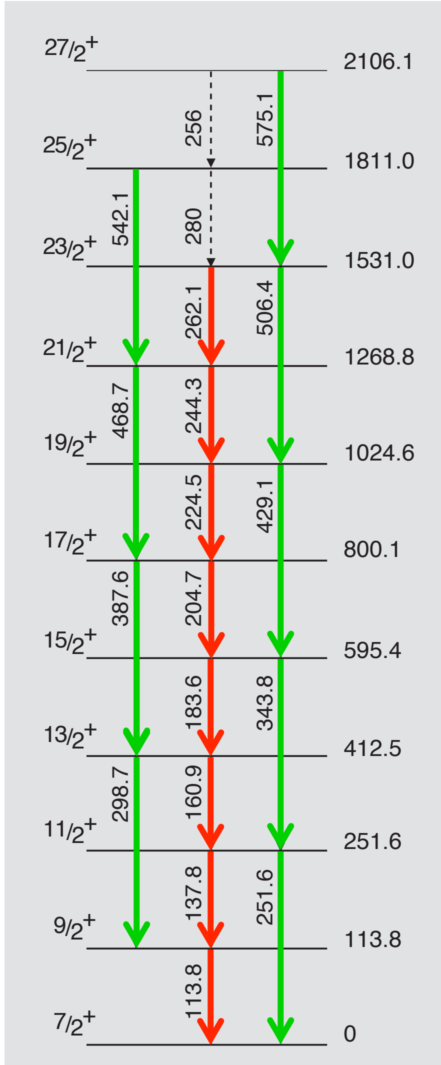


Figure: W.D. Kulp, Ga Tech

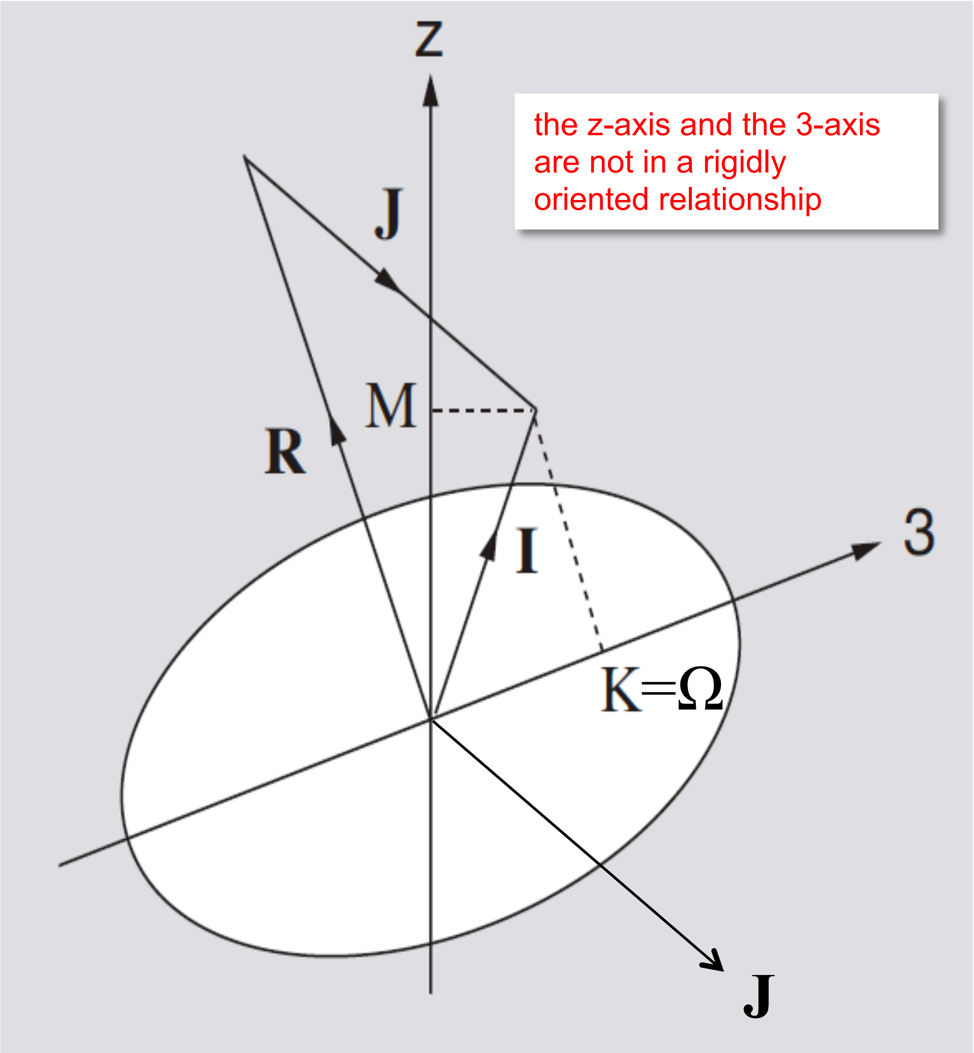
^{175}Lu: Coulex--^{40}Ca; “strong coupling”

Skensved et al., NP **A366** 125 1981





Symmetric-top model:
quantum numbers



R: collective angular momentum

J: intrinsic spin

I: total spin / angular momentum

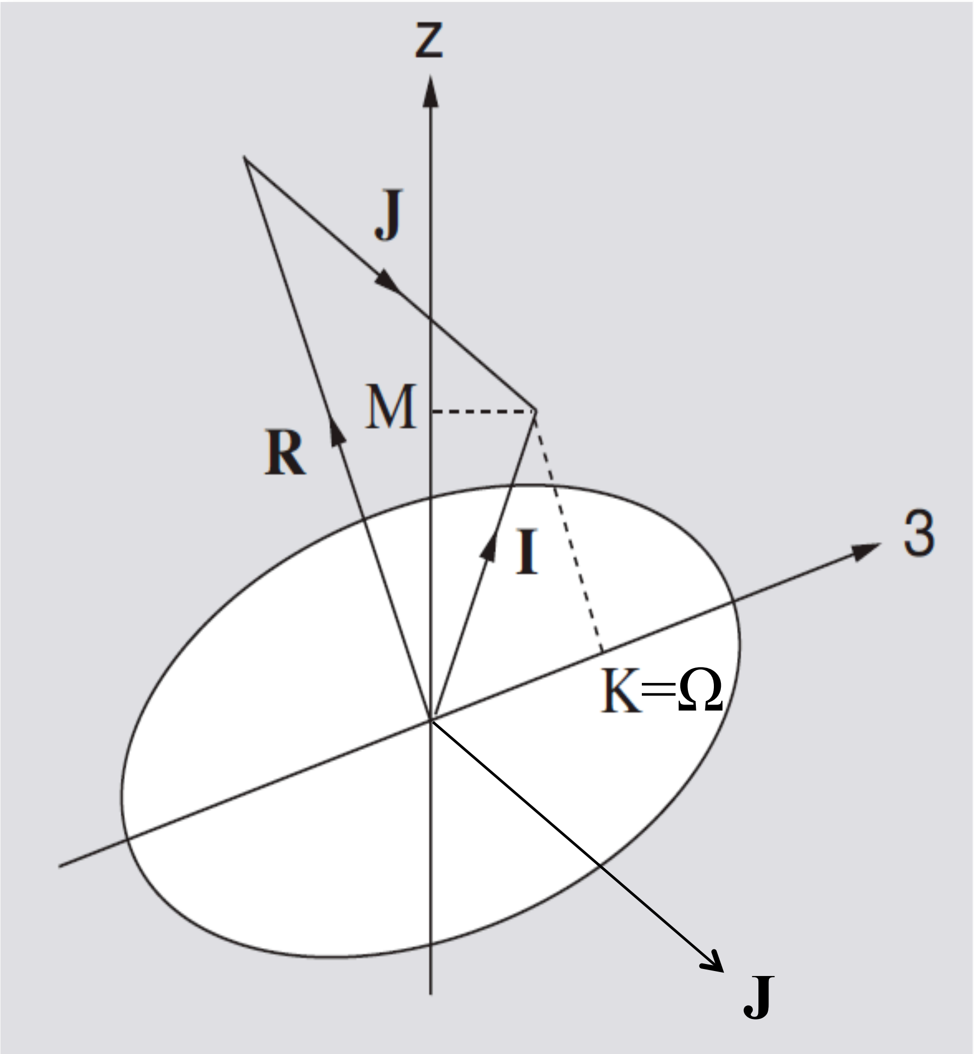
M: laboratory-frame,
z-component of I

K: body-frame (symmetry axis),
3-component of I; K = Ω

R&W Fig. 1.46

Symmetric-top model:

rotational-particle coupling, RPC



$$\hat{H}_{\text{rot}} = \frac{\hbar^2\,\hat{\mathbf{R}}^2}{2\Im}$$

RPC term:

$$\mathbf{R}\cdot\mathbf{R} = (\mathbf{I}-\mathbf{J})\cdot(\mathbf{I}-\mathbf{J})$$
$$= \mathbf{I}\cdot\mathbf{I} - 2\,\mathbf{I}\cdot\mathbf{J} + \mathbf{J}\cdot\mathbf{J}$$

R&W Fig. 1.46

The key concept for modeling deformed nuclei: the symmetric-top + Nilsson model

INTRODUCTION

$$\hat{H} := \frac{\hbar^2 \hat{\mathbf{R}}^2}{2\Im} + \boxed{\hat{h}},$$

"coupling" = add Hamiltonians
(wave functions will be direct products)

$$\boxed{\hat{h} := \frac{\hat{p}^2}{2M} + \tfrac{1}{2} M \left[\omega_\perp^2 (\bar{x}^2 + \bar{y}^2) + \omega_z^2 \bar{z}^2\right] + D \hat{\mathbf{l}}^2 + \xi\, \hat{\mathbf{l}} \cdot \hat{\mathbf{s}}.}$$

$$\hat{\mathbf{I}} := \hat{\mathbf{R}} + \hat{\mathbf{j}}$$

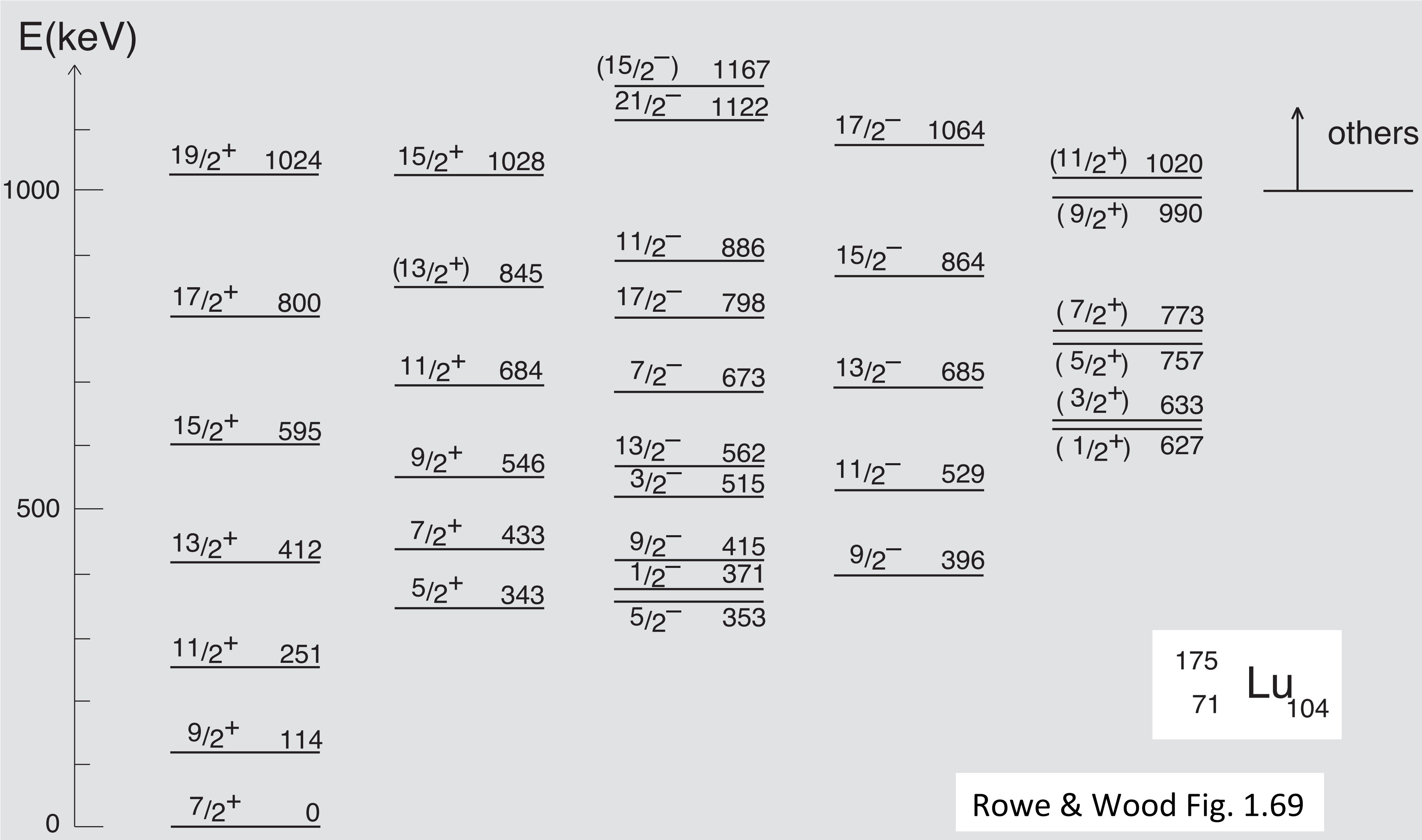
"coupling" = add spins/angular momenta

$$\hat{H} = \frac{\hbar^2 \hat{\mathbf{I}}^2}{2\Im} + \boxed{\hat{h}} + \frac{\hbar^2 \hat{\mathbf{j}}^2}{2\Im} \boxed{- \frac{\hbar^2}{\Im} \hat{\mathbf{I}} \cdot \hat{\mathbf{j}}}$$

FOCUS, THIS TALK

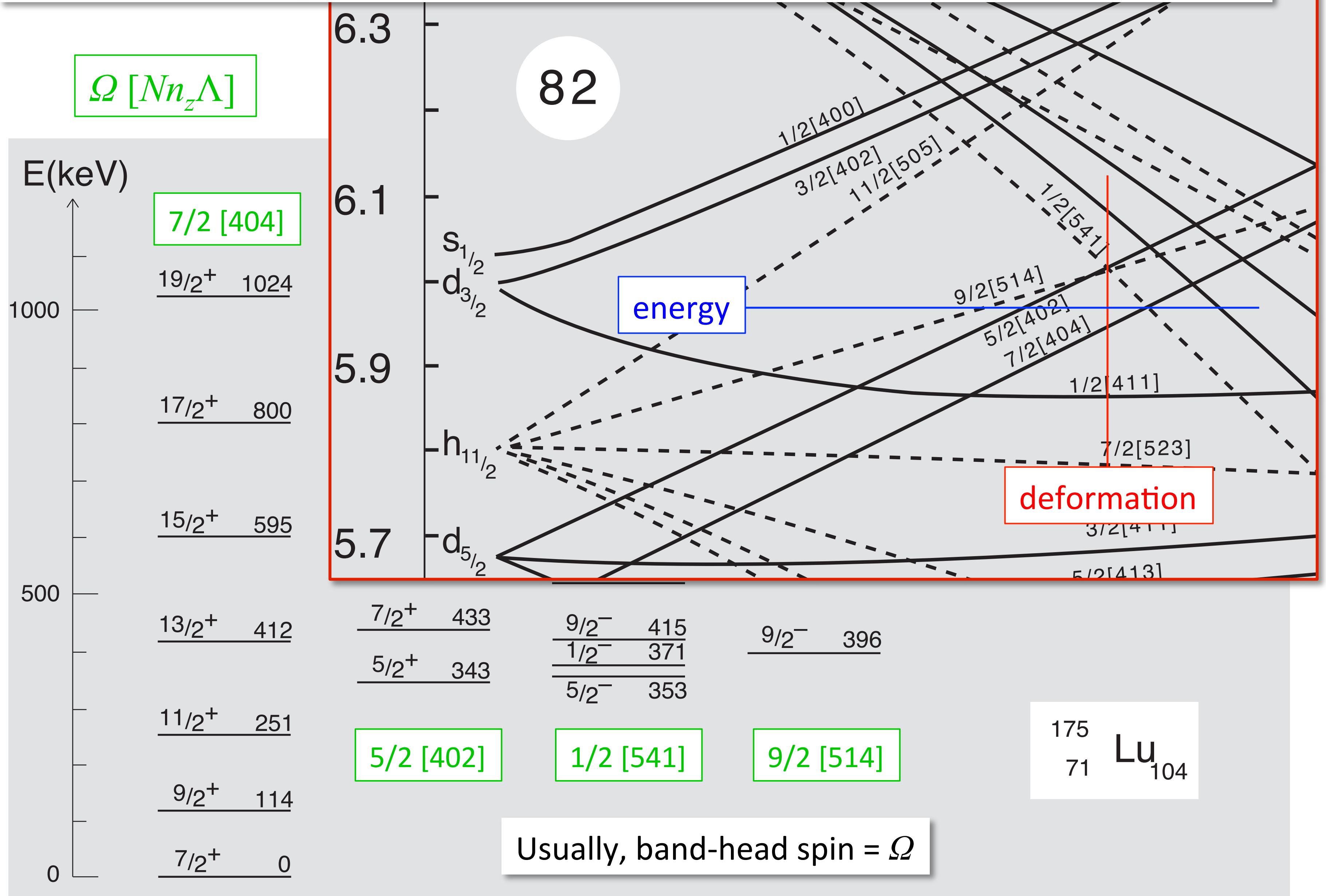
Symmetric-top-plus-Nilsson model in ^{175}Lu:

organizes 34 states into 5 rotational bands

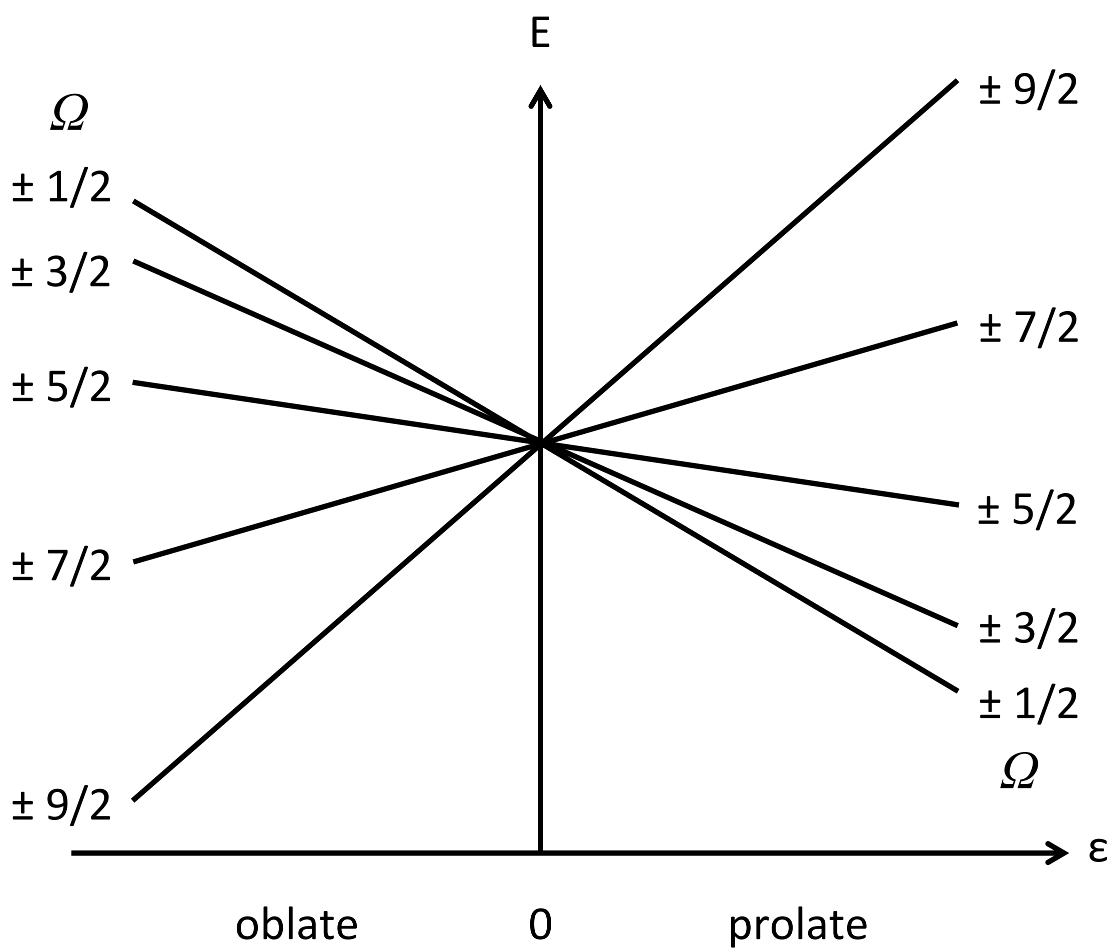


Rowe & Wood Fig. 1.69

Nilsson diagram and identification of band heads



Elementary quantum mechanics of the Nilsson model--1

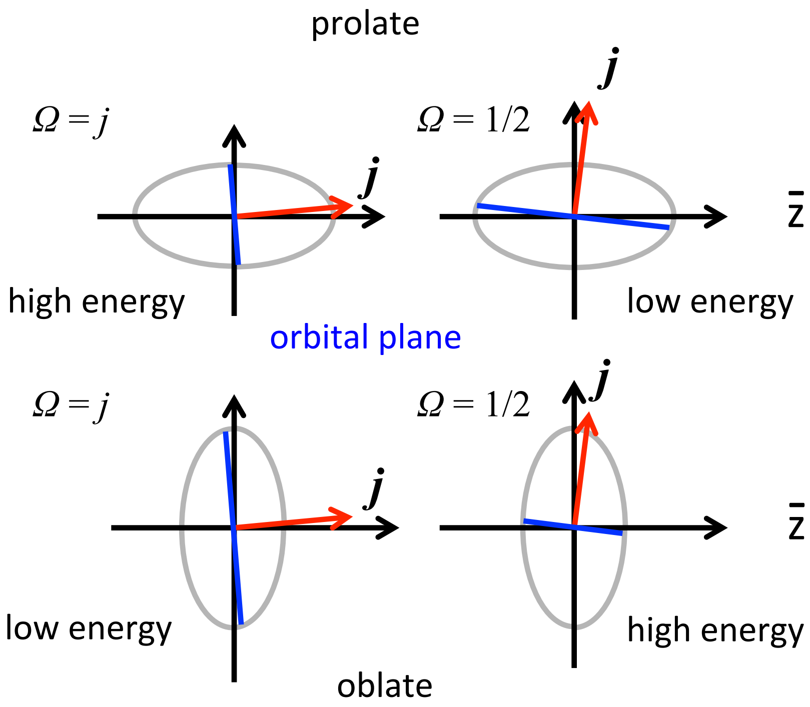


Nilsson diagram for a j = 9/2 spherical configuration

E ~ ε Ω^2

Reflection symmetry of the potential leads to ± Ω degeneracy

Elementary quantum mechanics of the Nilsson model--2



Anisotropy of the harmonic oscillator potential results in "narrow" confinement (high energy) or "wide" confinement (low energy): "deformation alignment" --ordering dictated by Pauli principle.

Extreme prolate deformation: n_z quanta much lower energy than n_x, n_y quanta—n_z becomes a good "asymptotic" quantum number.

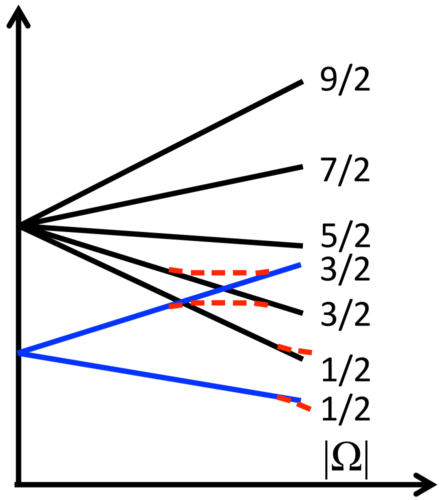
Nilsson quantum numbers, $\Omega^\pi\, [N\, n_z\, \Lambda\, \Sigma]$:

$\Omega = \Lambda + \Sigma$; $\quad \Omega = j_z,\ \Lambda = l_z,\ \Sigma = s_z$;

$\pi = (-1)^N$; $\quad \Sigma = \uparrow$ or $\downarrow$

$$\hat{h} := \frac{\hat{p}^2}{2M} + \tfrac{1}{2} M \left[\omega_\perp^2 (\bar{x}^2 + \bar{y}^2) + \omega_z^2 \bar{z}^2\right] + D\hat{\mathbf{l}}^2 + \xi\, \hat{\mathbf{l}} \cdot \hat{\mathbf{s}}$$

Elementary quantum mechanics of the Nilsson model--3



Spherical basis

$H_{mix} \sim r^2 Y_{20}$

$$|\Omega\rangle = \sum_{Nlj} c_{lj} |Nlj\Omega\rangle$$

Mixing produced by H_{mix}:

$\Delta l = \pm\, 0, 2, 4, \ldots$

$\Delta N = \pm\, 0, 2, \ldots$

$\Delta\Omega = 0$ only,

(recall, $\pi = (-1)^N$ and for
N even, only even l;
N odd, only odd l)

there is no parity mixing
and no effect on nucleon spin*.

*To calculate matrix elements, uncouple j into l and s, work in the l-basis, then recouple l with s to obtain j.

Nilsson model plus rotations

$$\hat{H} = \frac{\hbar^2 \hat{\mathbf{I}}^2}{2\Im} + \quad h_{\text{Nilsson}} \boxed{+ \frac{\hbar^2 \hat{\mathbf{j}}^2}{2\Im} - \frac{\hbar^2}{\Im}\hat{\mathbf{I}} \cdot \hat{\mathbf{j}}.}$$

"Coriolis":
"recoil" + RPC terms

$$\langle \vec{I} \cdot \vec{j} \rangle = \langle \frac{1}{2}(\hat{I}_+ \hat{j}_- + \hat{I}_- \hat{j}_+) + \hat{I}_z \hat{j}_z \rangle$$

$I_+ := I_x + iI_y$, $I_- = I_x - iI_y$
$j_+ := j_x + ij_y$, $j_- = j_x - ij_y$

$$|KIM\rangle + \varepsilon(-1)^{I+K}| -K, IM\rangle$$

ε = +1 reflection symmetric
ε = -1 reflection asymmetric

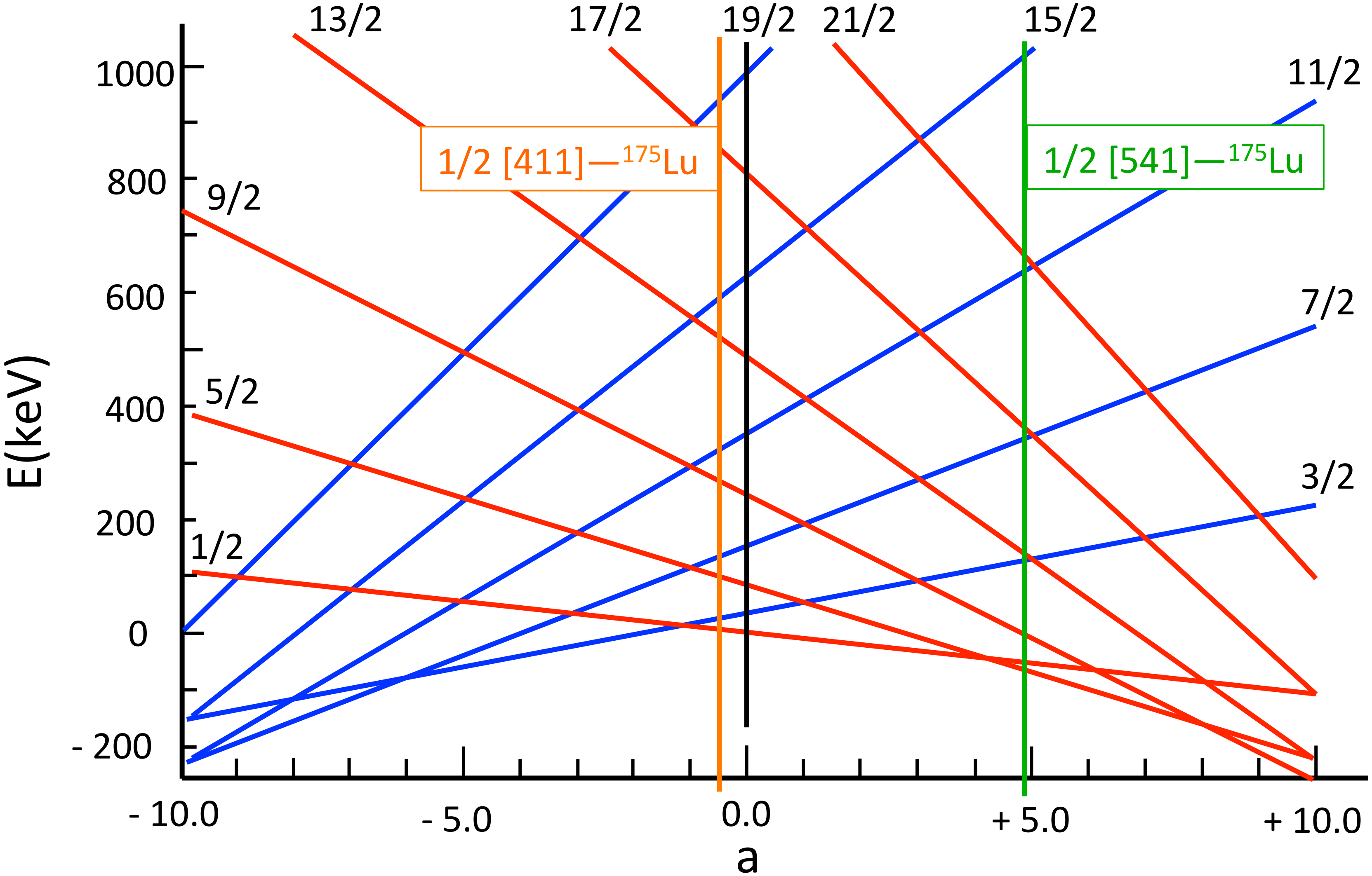
$$E_I = E_0 + A[I(I+1) + (-1)^{I+1/2}(I + 1/2)\, a\, \delta_{K,1/2}]$$

$\delta_{K,1/2} = 1,\ K = ½$
$\delta_{K,1/2} = 0$ otherwise

Quantum mechanics of particle-rotor coupling: K = ½ bands and "Coriolis" decoupling

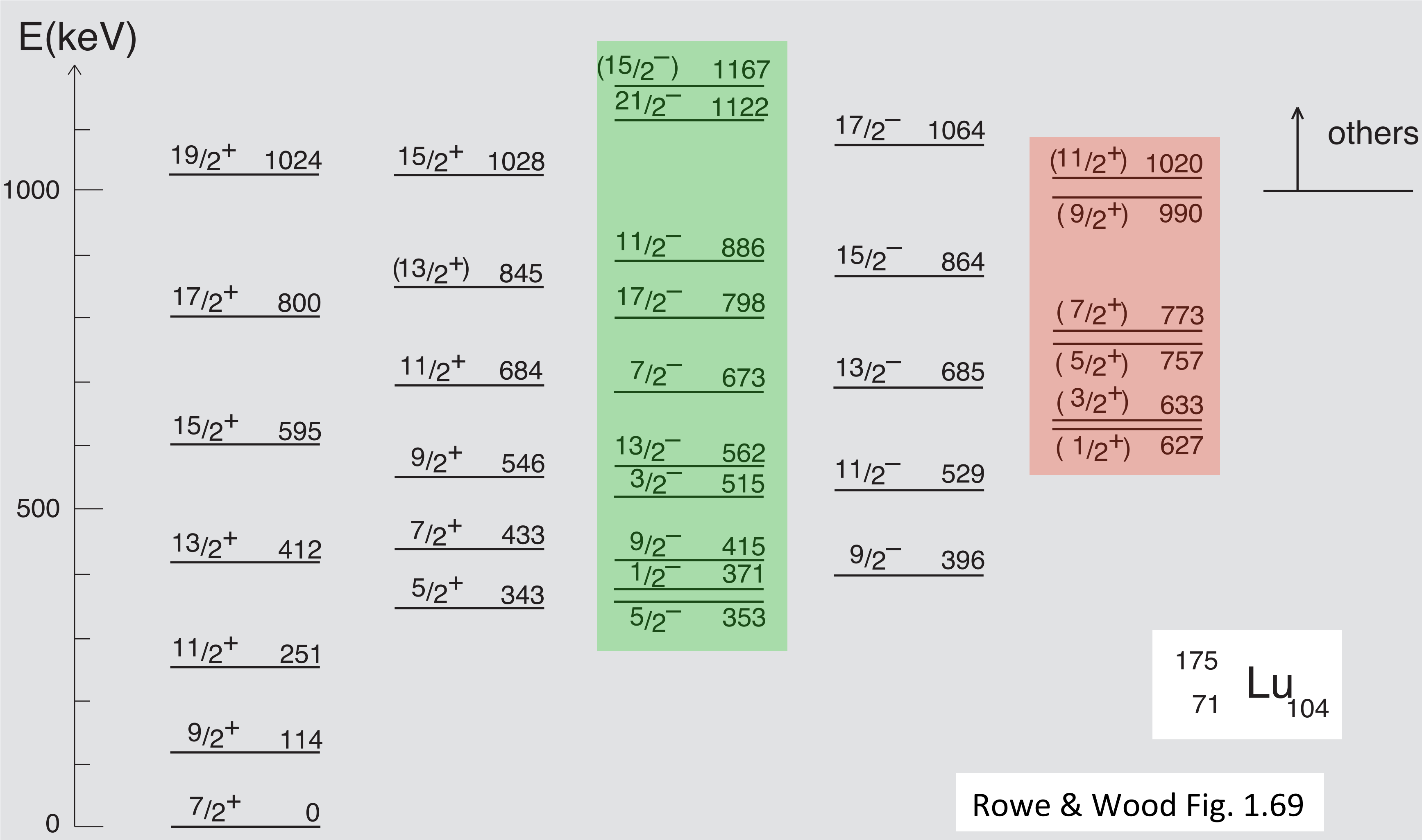
$$E_I = E_0 + A[I(I+1) + (-1)^{I+1/2}(I + 1/2)\, a\, \delta_{K,1/2}]$$

$A = 10.0$ keV



Symmetric-top-plus-Nilsson model in ^{175}Lu:

organizes 34 states into 5 rotational bands



Rowe & Wood Fig. 1.69

Quantum mechanics of particle-rotor coupling: K = ½ bands and “Coriolis” decoupling

$$E_I = E_0 + A[I(I+1) + (-1)^{I+1/2}(I+1/2)\, a\, \delta_{K,1/2}]$$

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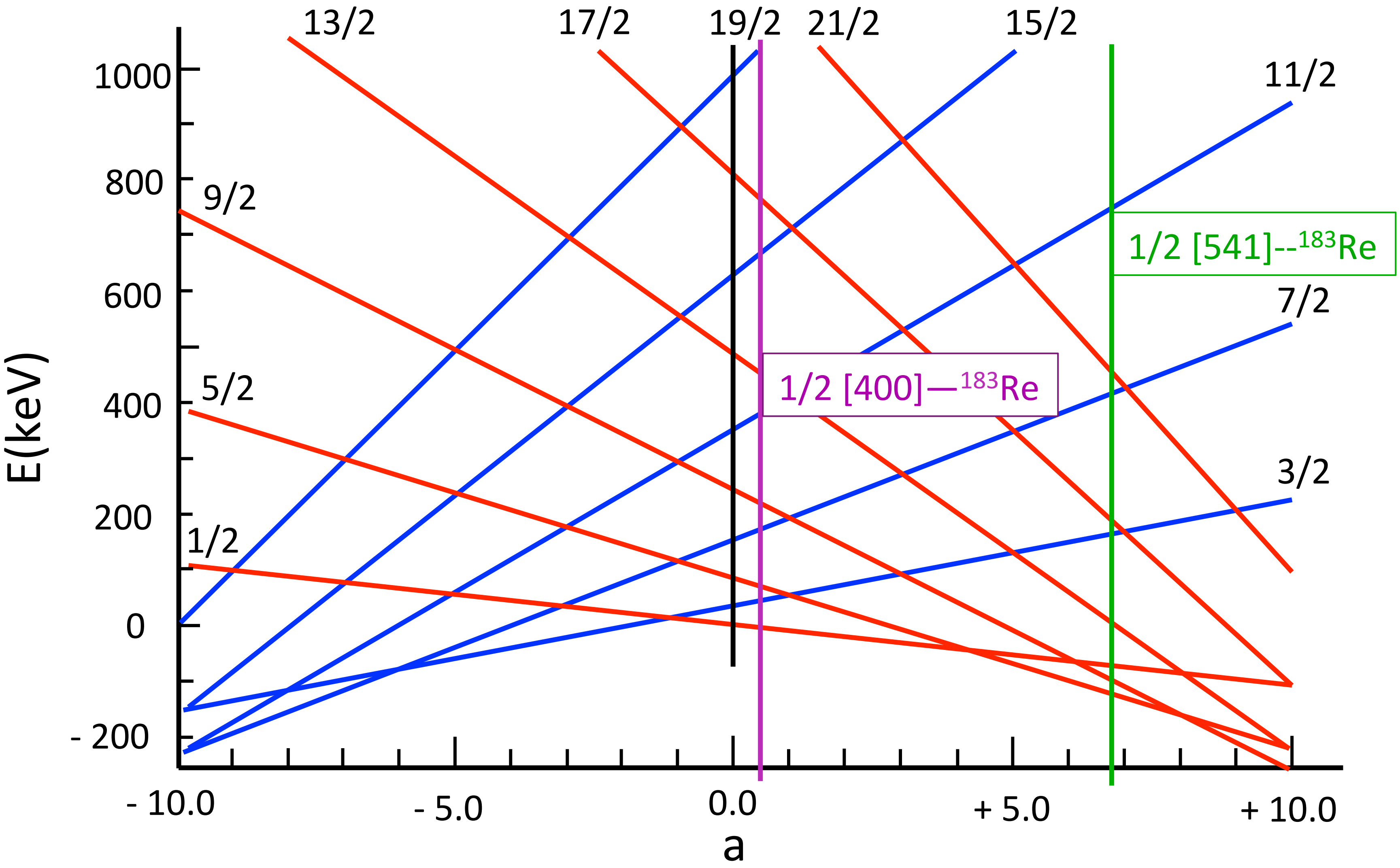
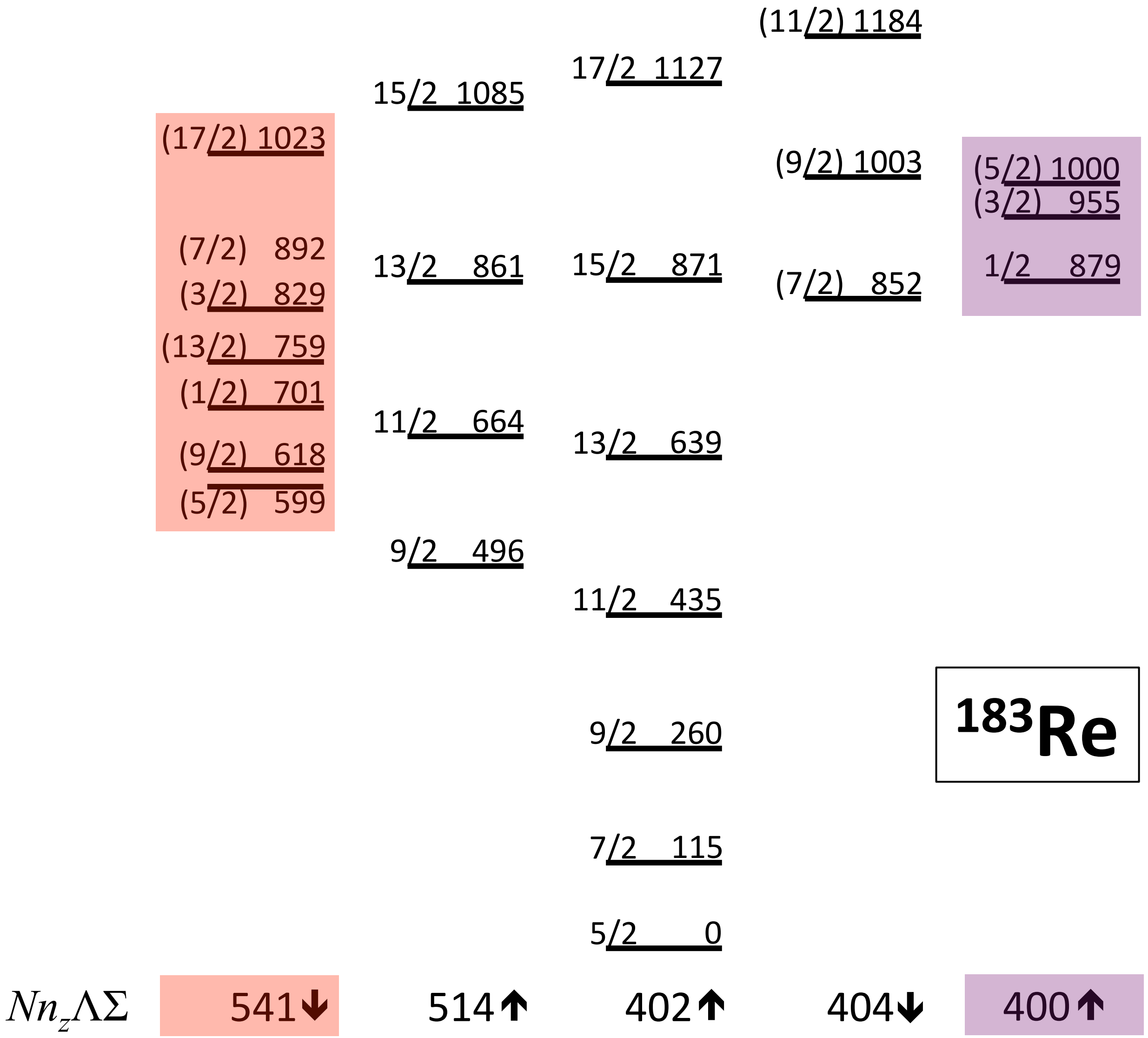
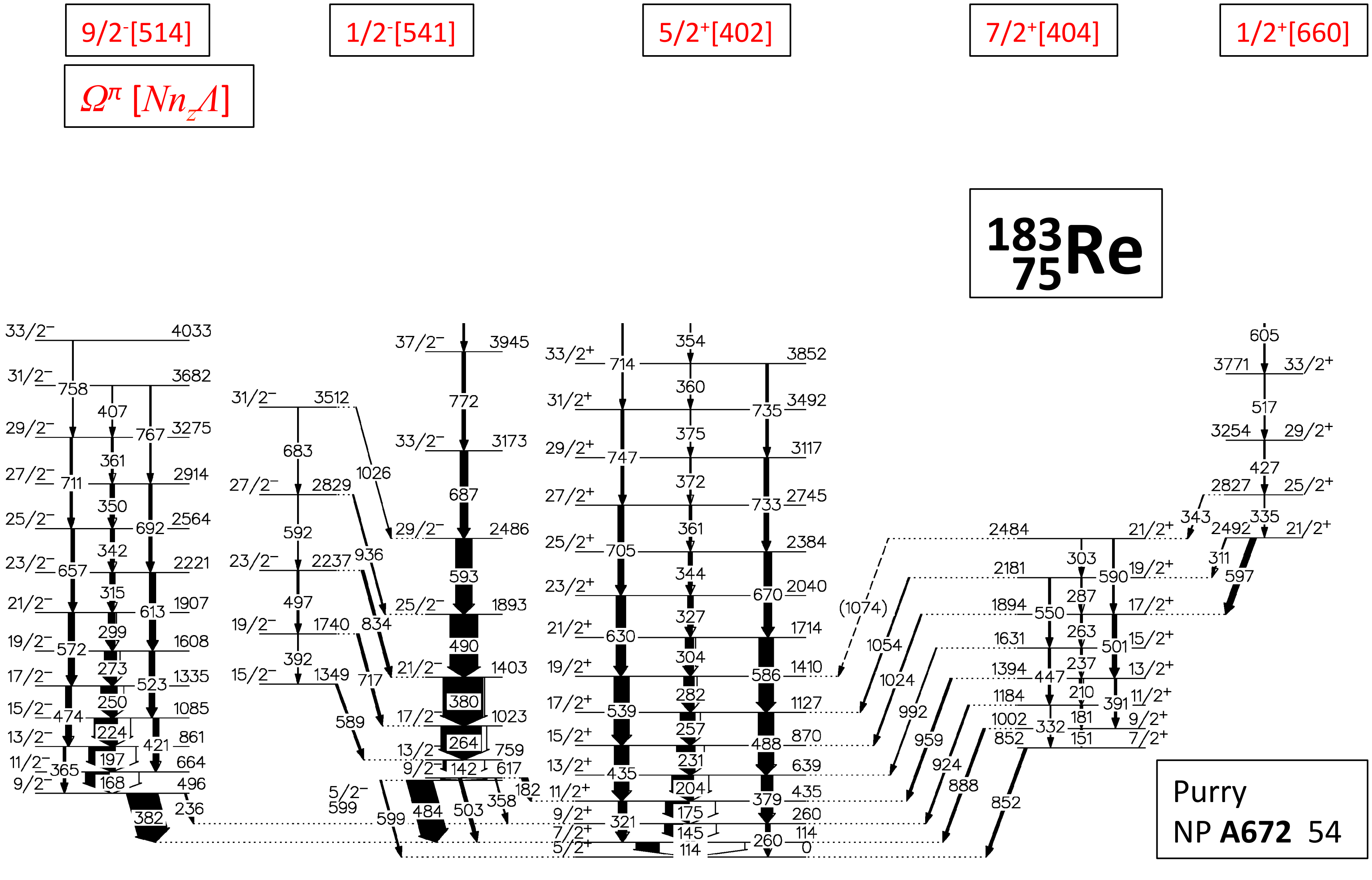




(11/2) 1184
17/2 1127
15/2 1085
(17/2) 1023
(9/2) 1003
(5/2) 1000
(3/2) 955
(7/2) 892
1/2 879
15/2 871
13/2 861
(7/2) 852
(3/2) 829
(13/2) 759
(1/2) 701
11/2 664
13/2 639
(9/2) 618
(5/2) 599
9/2 496
11/2 435
^{183}Re
9/2 260
7/2 115
5/2 0
$Nn_z\Lambda\Sigma$
541↓
514↑
402↑
404↓
400↑

“Strong coupling” of single nucleon to deformed core: Nilsson model plus symmetric-top model



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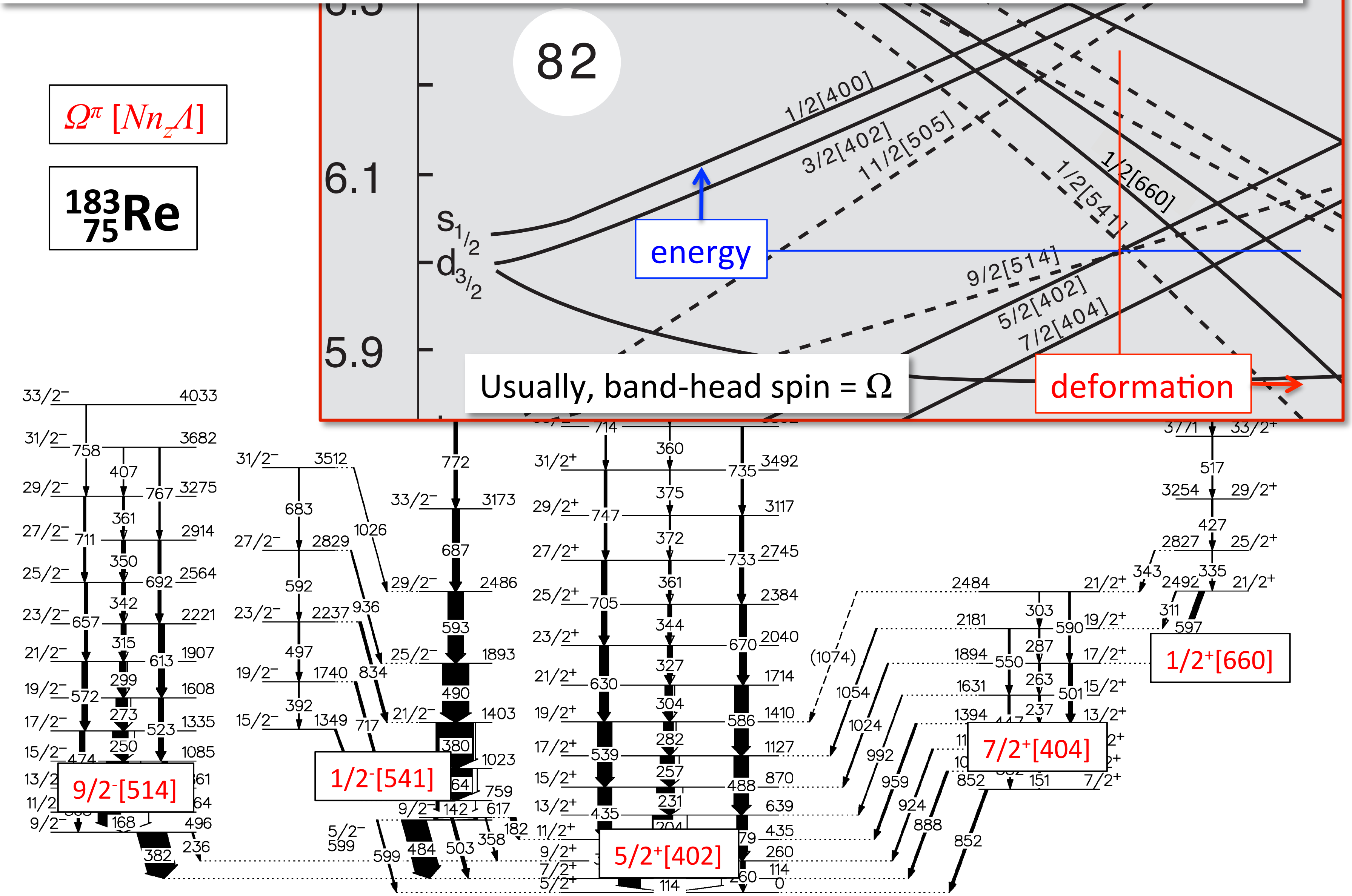
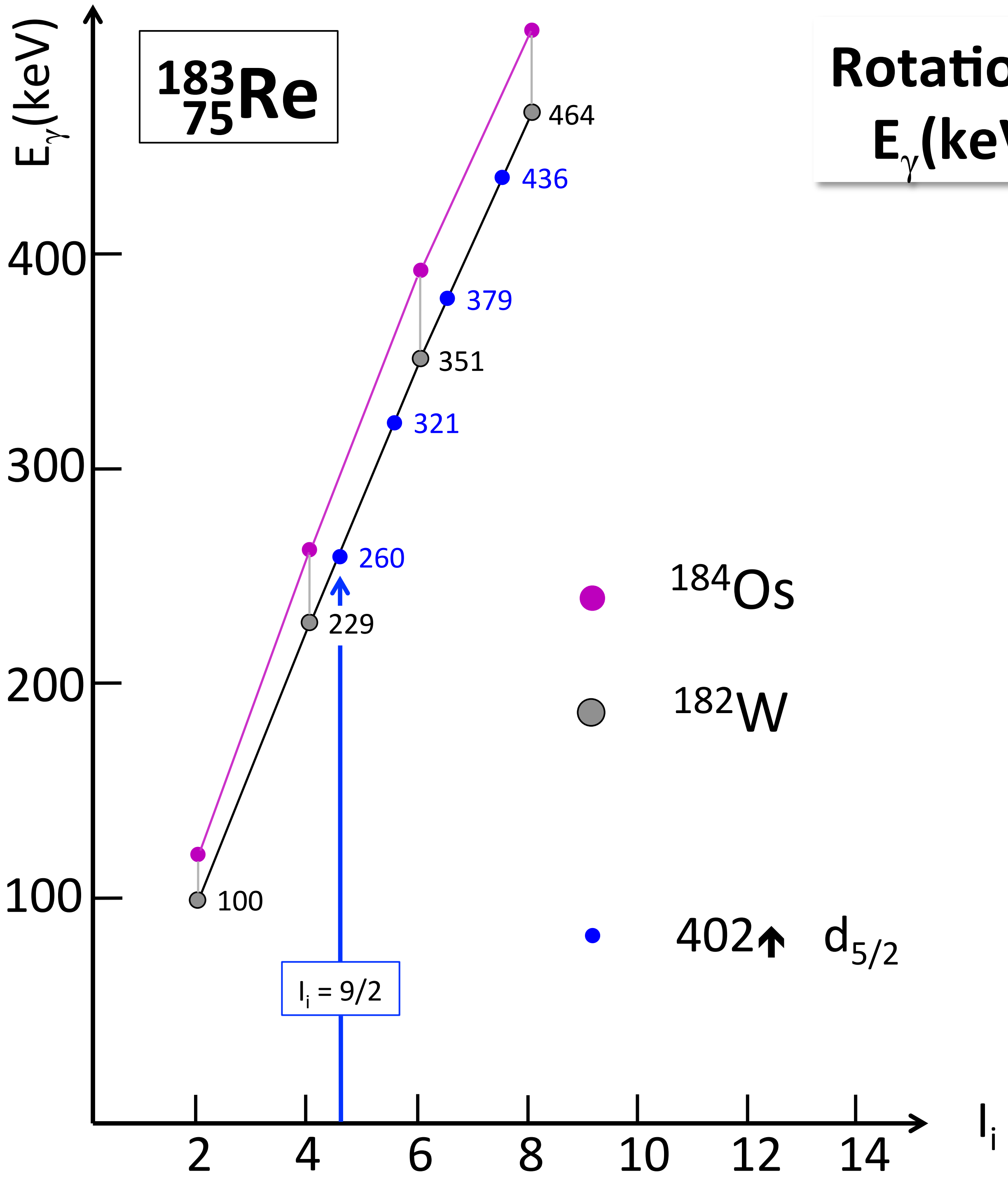
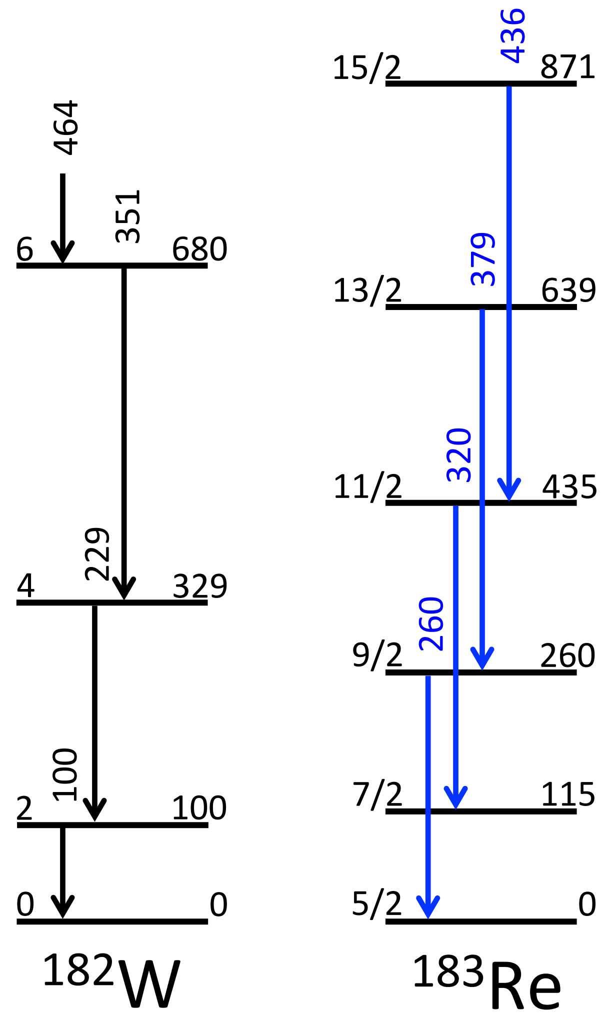


Rotation alignment plots— E_γ(keV) vs. I_i

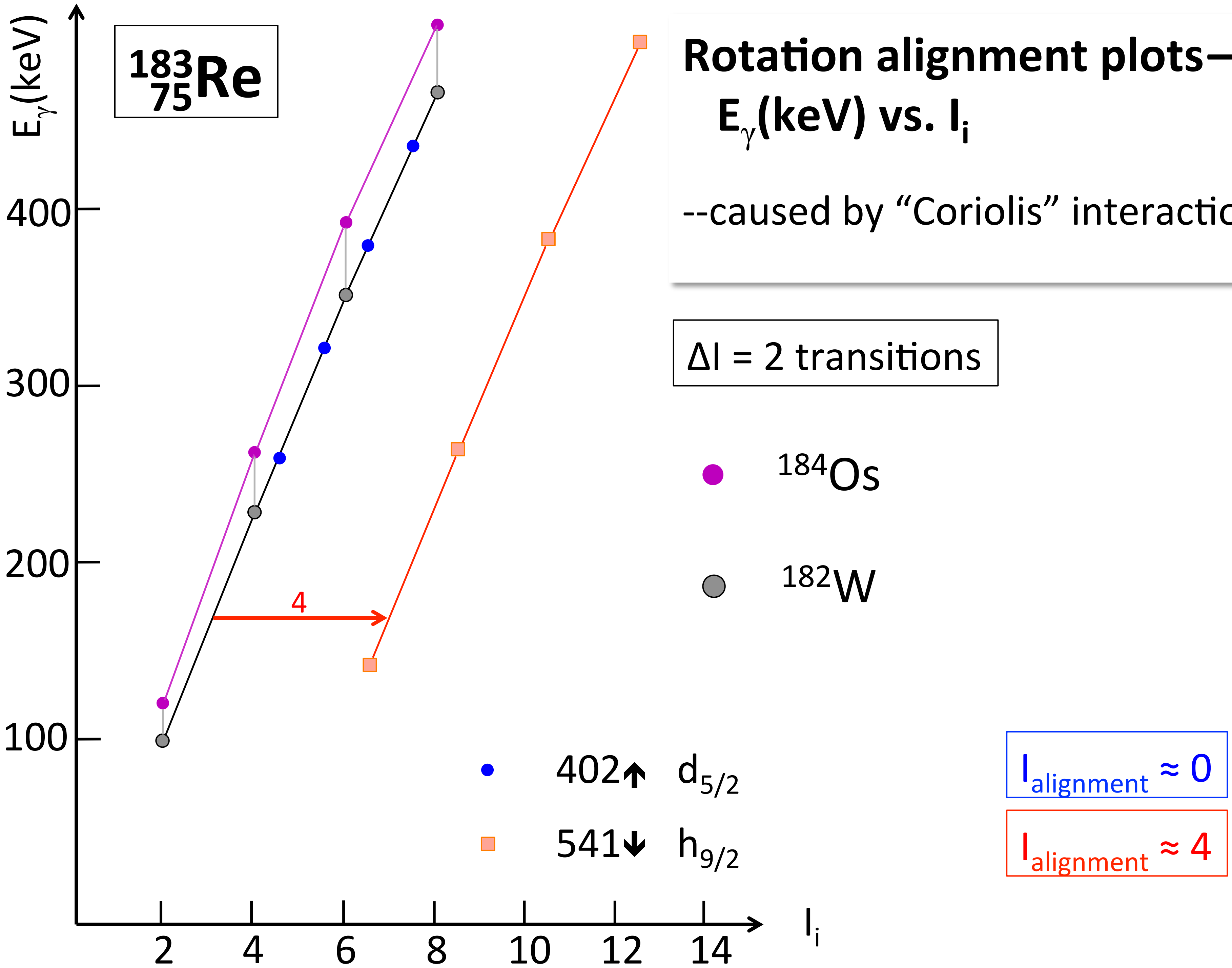
ΔI = 2 transitions

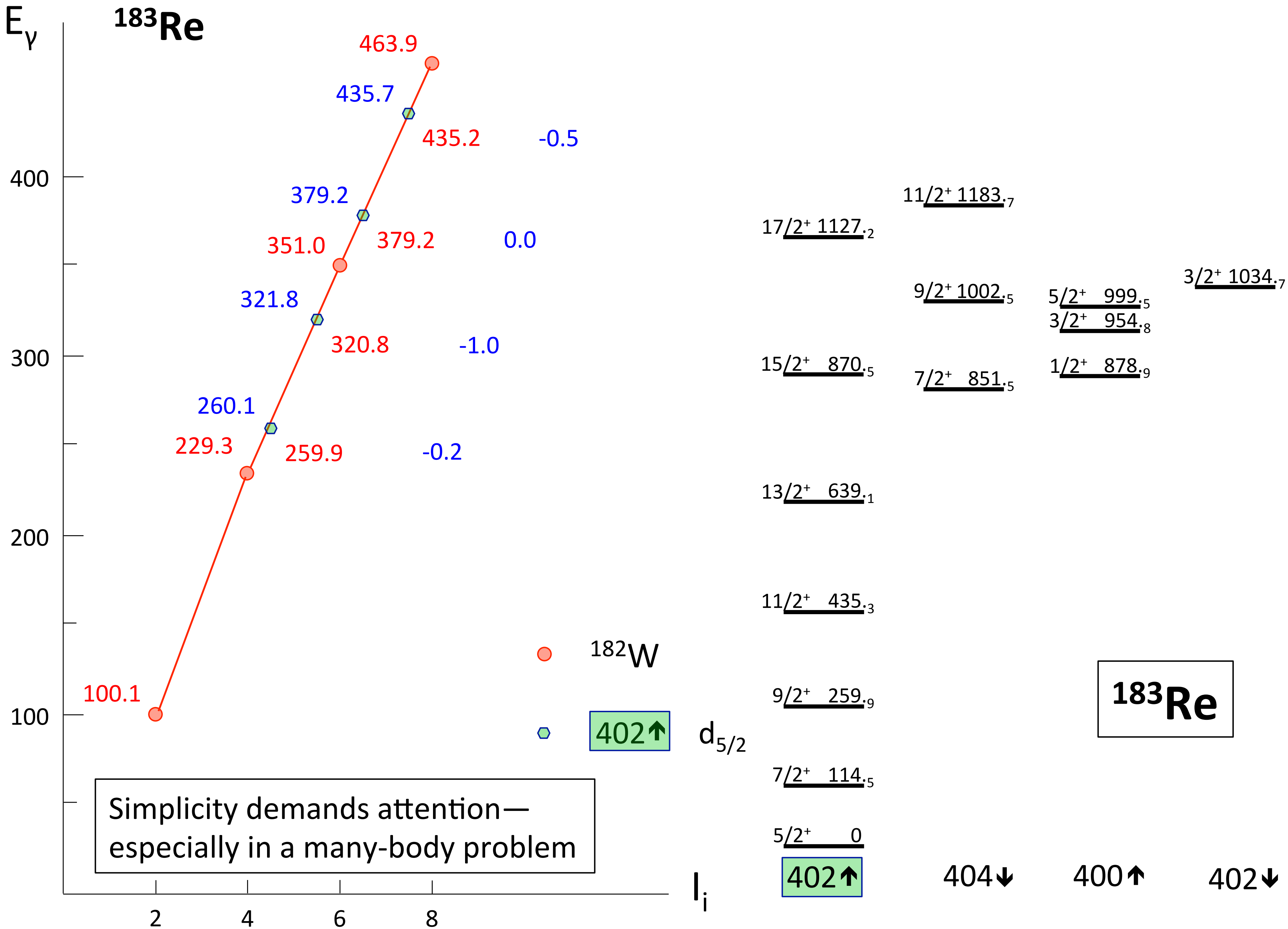




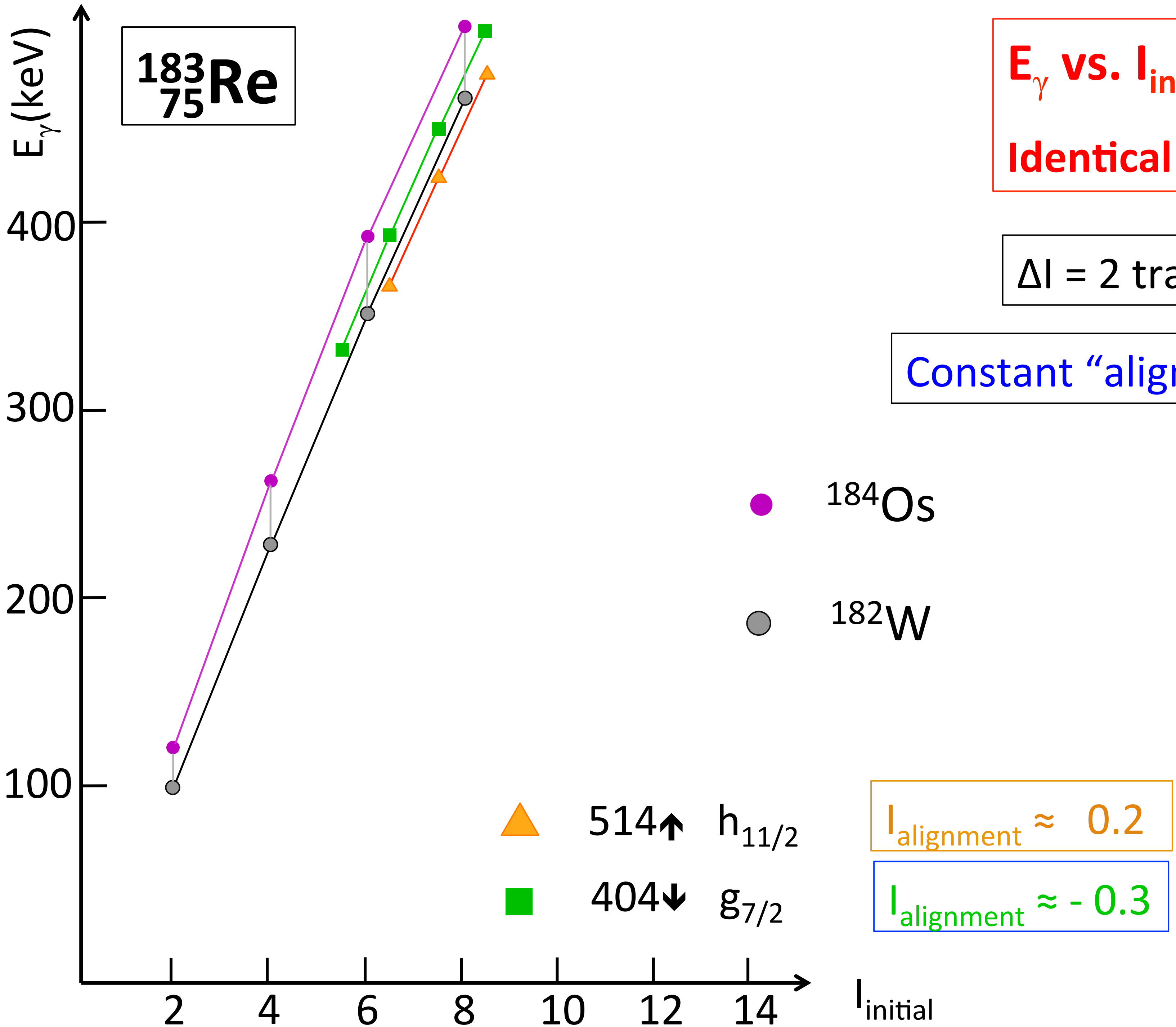
Rotation alignment plots—
E_γ(keV) vs. I_i

--caused by “Coriolis” interaction





E_γ
^{183}Re
463.9
435.7
435.2
-0.5
400
379.2
379.2
351.0
0.0
321.8
320.8
-1.0
300
260.1
229.3
259.9
-0.2
200
^{182}W
100.1
100
402↑
$d_{5/2}$
Simplicity demands attention—
especially in a many-body problem
2
4
6
8
I_i
11/2+ 1183.7
17/2+ 1127.2
9/2+ 1002.5
5/2+ 999.5
3/2+ 1034.7
3/2+ 954.8
15/2+ 870.5
7/2+ 851.5
1/2+ 878.9
13/2+ 639.1
11/2+ 435.3
9/2+ 259.9
7/2+ 114.5
5/2+ 0
^{183}Re
402↑
404↓
400↑
402↓

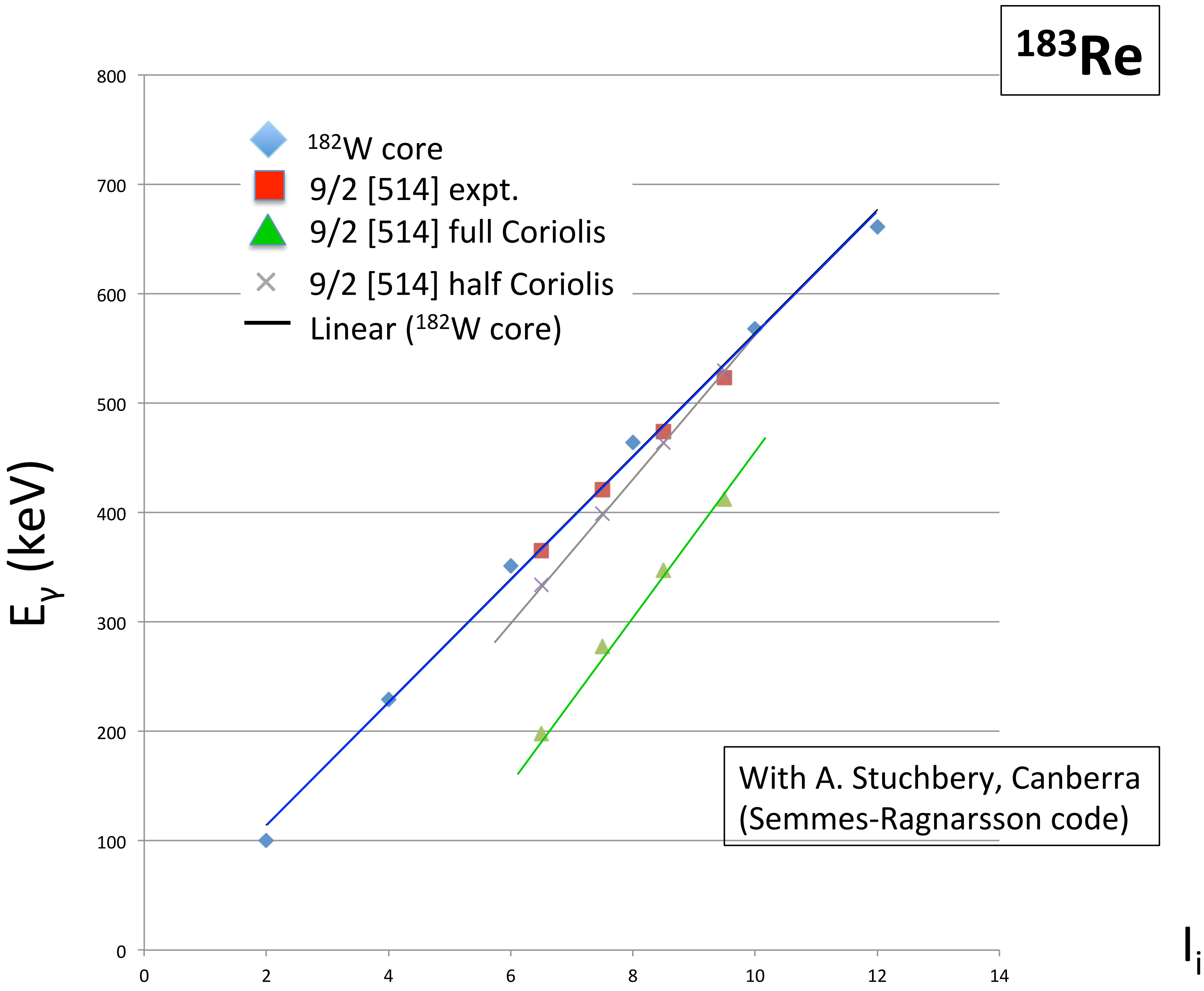


E_γ vs. $I_{initial}$ plot

Identical bands

ΔI = 2 transitions

Constant "alignments"



183Re
182W core
9/2 [514] expt.
9/2 [514] full Coriolis
9/2 [514] half Coriolis
Linear (182W core)
With A. Stuchbery, Canberra
(Semmes-Ragnarsson code)
Eγ (keV)
Ii
0
100
200
300
400
500
600
700
800
0
2
4
6
8
10
12
14

The key concept for modeling deformed nuclei: the symmetric-top + Nilsson model

$$\hat{H} := \frac{\hbar^2 \hat{\mathbf{R}}^2}{2\Im} + \hat{h},$$

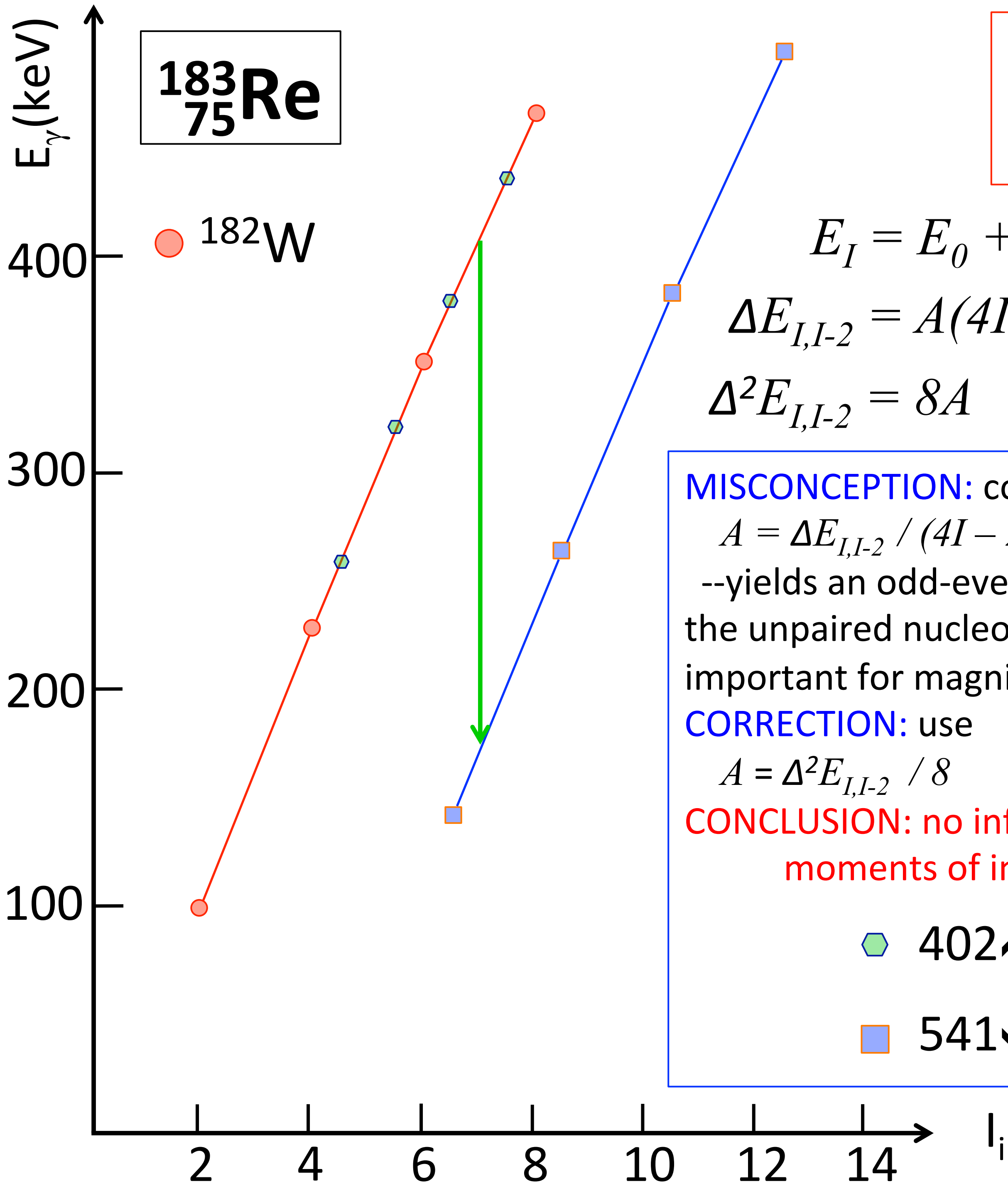
"coupling" = add Hamiltonians
(wave functions will be direct products)

$$\hat{h} := \frac{\hat{p}^2}{2M} + \tfrac{1}{2} M \left[\omega_\perp^2 (\bar{x}^2 + \bar{y}^2) + \omega_z^2 \bar{z}^2\right] + D\hat{\mathbf{l}}^2 + \xi\, \hat{\mathbf{l}} \cdot \hat{\mathbf{s}}.$$

$$\hat{\mathbf{I}} := \hat{\mathbf{R}} + \hat{\mathbf{j}}$$

"coupling" = add spins/angular momenta

$$\hat{H} = \frac{\hbar^2 \hat{\mathbf{I}}^2}{2\Im} + \hat{h} + \frac{\hbar^2 \hat{\mathbf{j}}^2}{2\Im} \boxed{- \frac{\hbar^2}{\Im} \hat{\mathbf{I}} \cdot \hat{\mathbf{j}}}$$



E_γ vs. $I_{initial}$ as a rotation alignment plot

$$E_I = E_0 + AI(I+1) - 2A\langle I \bullet J \rangle$$

$$\Delta E_{I,I-2} = A(4I-2) - const.$$

$$\Delta^2 E_{I,I-2} = 8A$$

MISCONCEPTION: commonly, use

$$A = \Delta E_{I,I-2} / (4I-2)$$

--yields an odd-even staggering and conclusion that the unpaired nucleon blocks pairing which is important for magnitudes of moments of inertia.

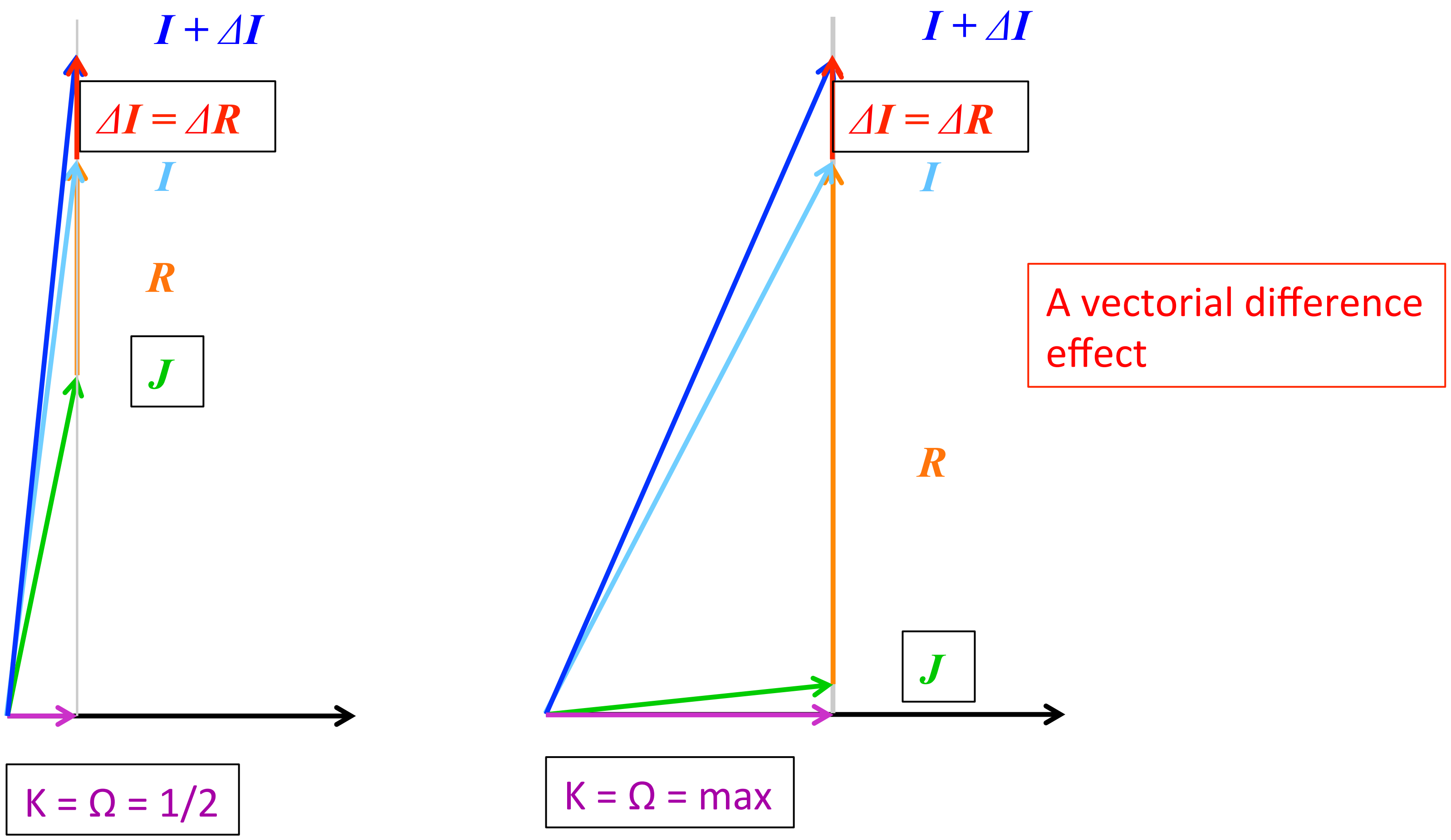
CORRECTION: use

$$A = \Delta^2 E_{I,I-2} / 8$$

CONCLUSION: no influence of pairing on moments of inertia

⬡	402↑	$d_{5/2}$	$const. = 0$ keV
■	541↓	$h_{9/2}$	$const. = -230$ keV

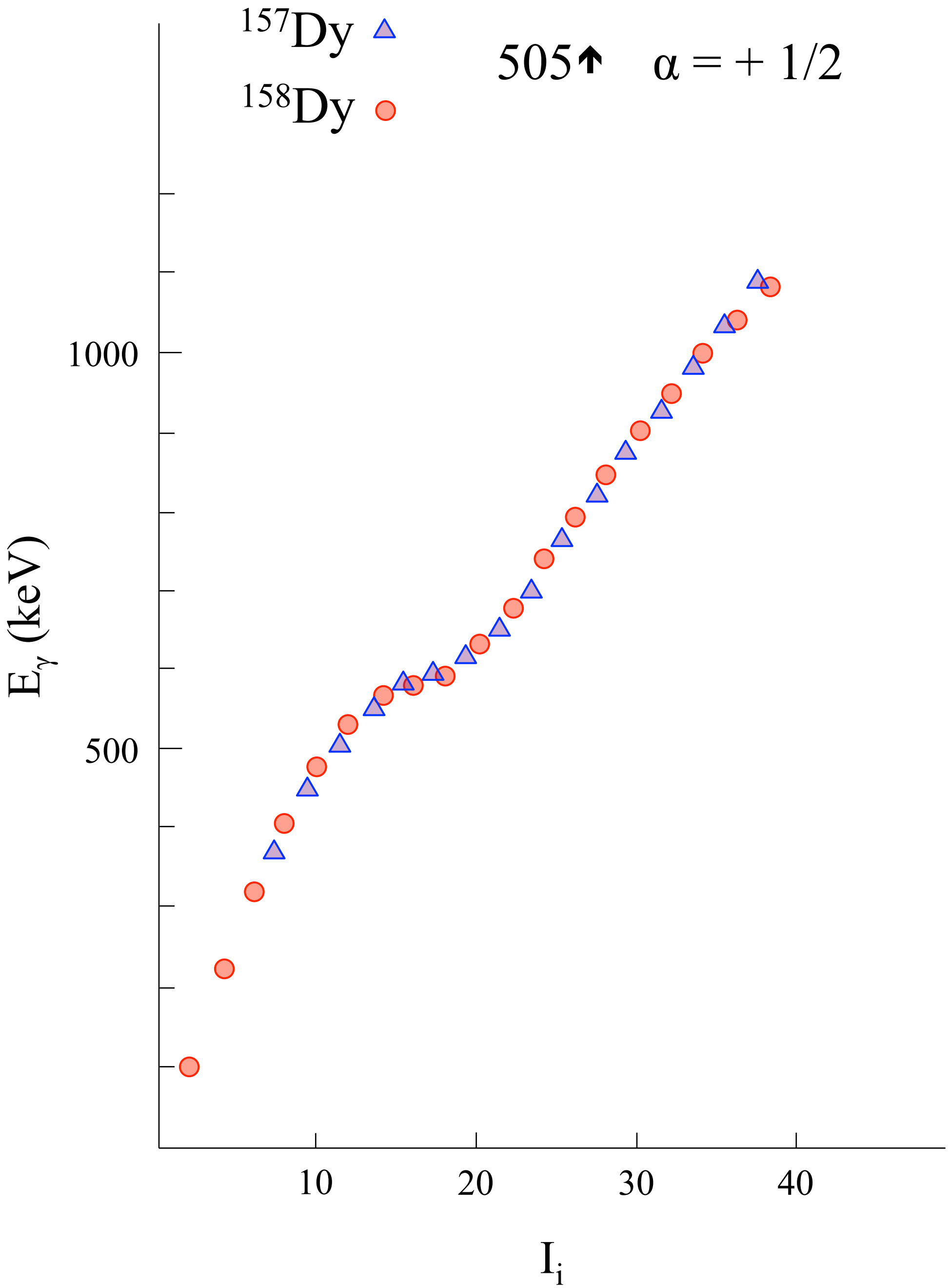
"Coriolis" contribution to energy *differences*



$\langle \Delta \boldsymbol{I} \bullet \boldsymbol{J} \rangle$ *large*

$\langle \Delta \boldsymbol{I} \bullet \boldsymbol{J} \rangle \approx 0$

Philosophical point: simple effects necessitate simple explanations



^{157}Dy

The "kink" in the profile is called backbending and will be a major focus in a later lecture.

CONCLUSIONS

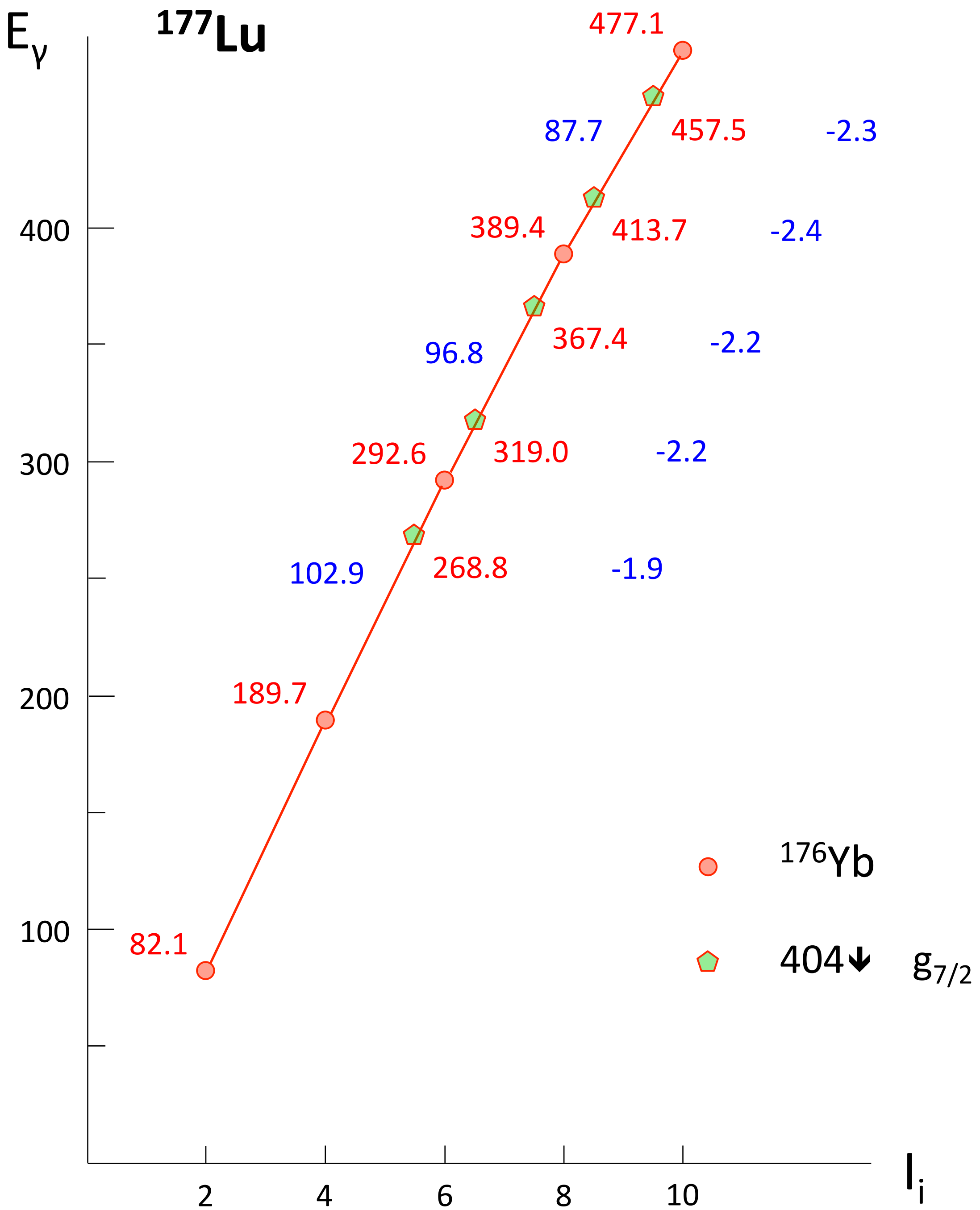
- Odd-mass nuclei exhibit bands that differ from the neighboring even-even nucleus ground-state bands by an "alignment" term described by **I•j**.

  Except for this difference, the bands are *identical**.

  CONSEQUENTLY, there is *no evidence* for:

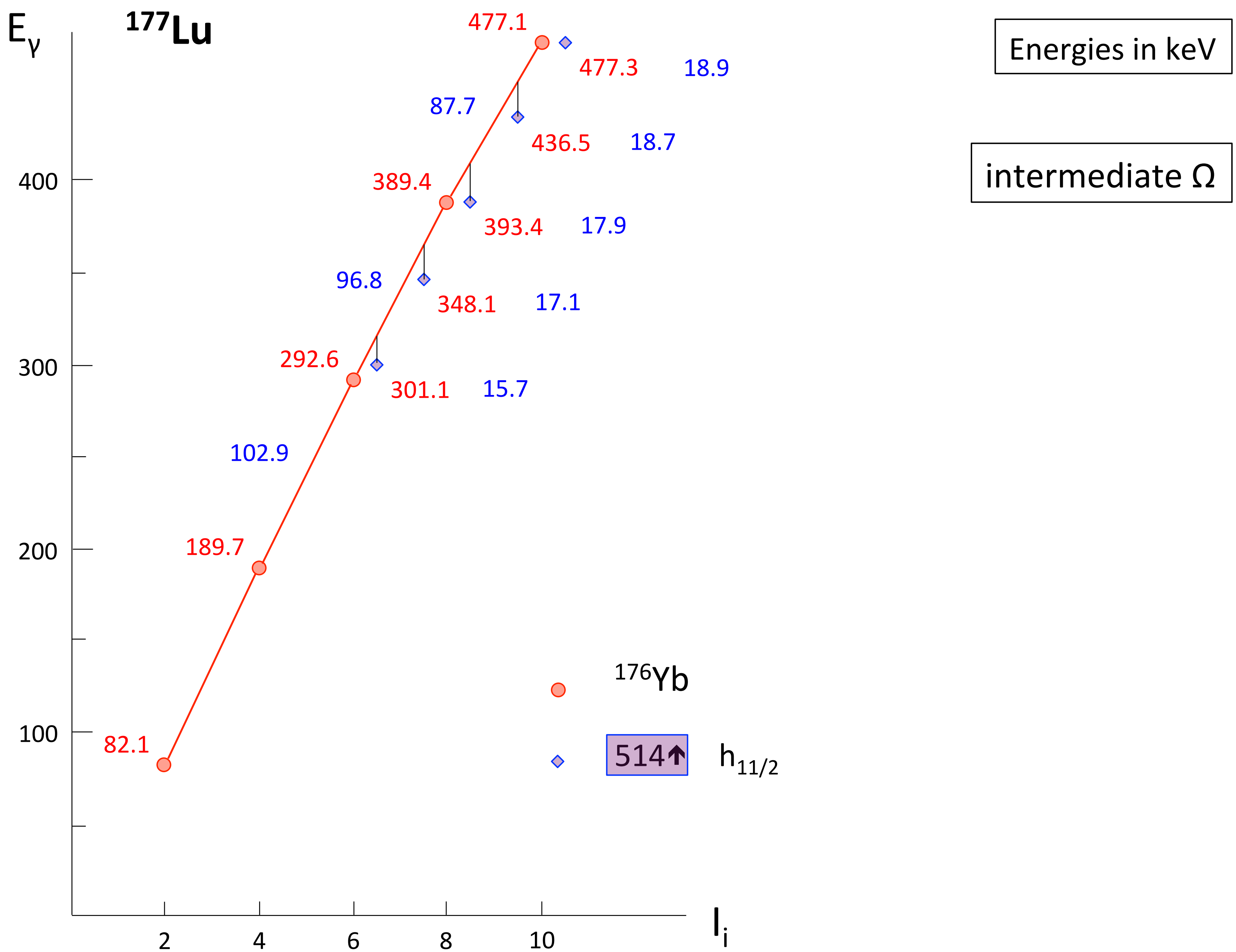
- odd-particle "blocking" of correlations involved in the even-even core collectivity;
- "deformation-driving" effects caused by the odd particle;
- "Coriolis" alignment effects (which should scale with increasing rotational frequency).

*There are small differences which can be attributed to band mixing.

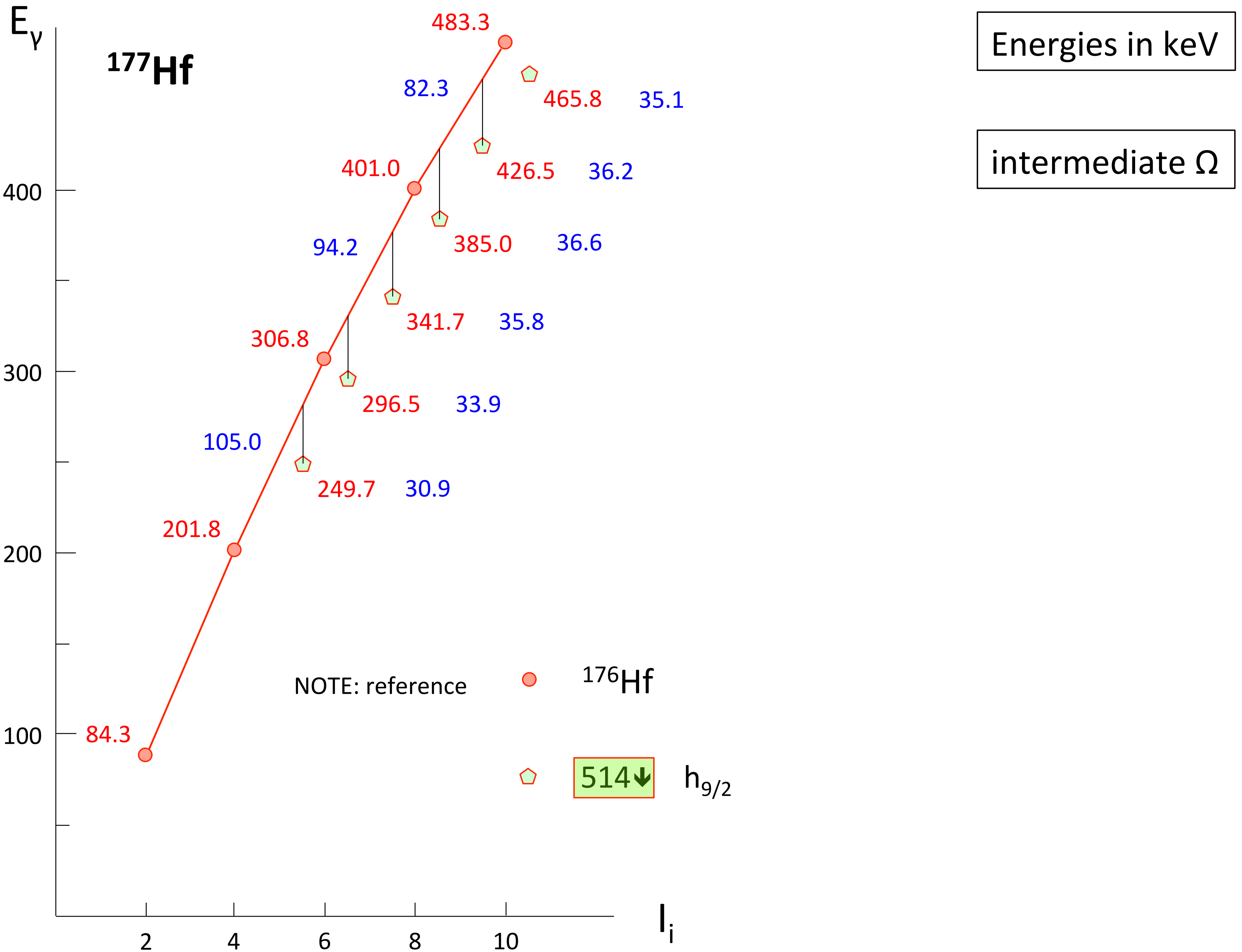


Energies in keV

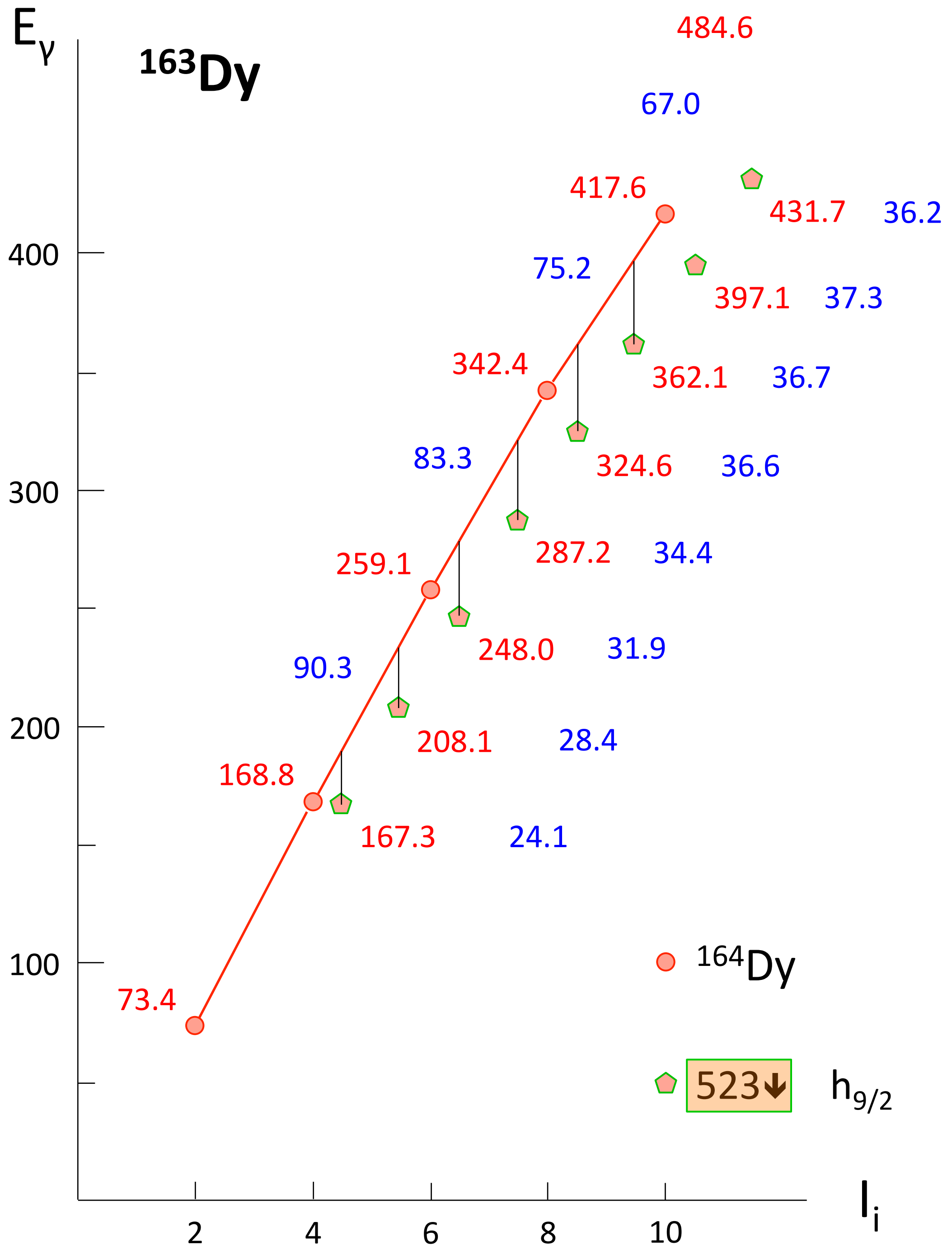
high Ω



E_γ
^{177}Lu
Energies in keV
intermediate Ω
477.1
477.3
18.9
87.7
436.5
18.7
389.4
393.4
17.9
96.8
348.1
17.1
292.6
301.1
15.7
102.9
189.7
82.1
400
300
200
100
^{176}Yb
514↑
$h_{11/2}$
2
4
6
8
10
I_i

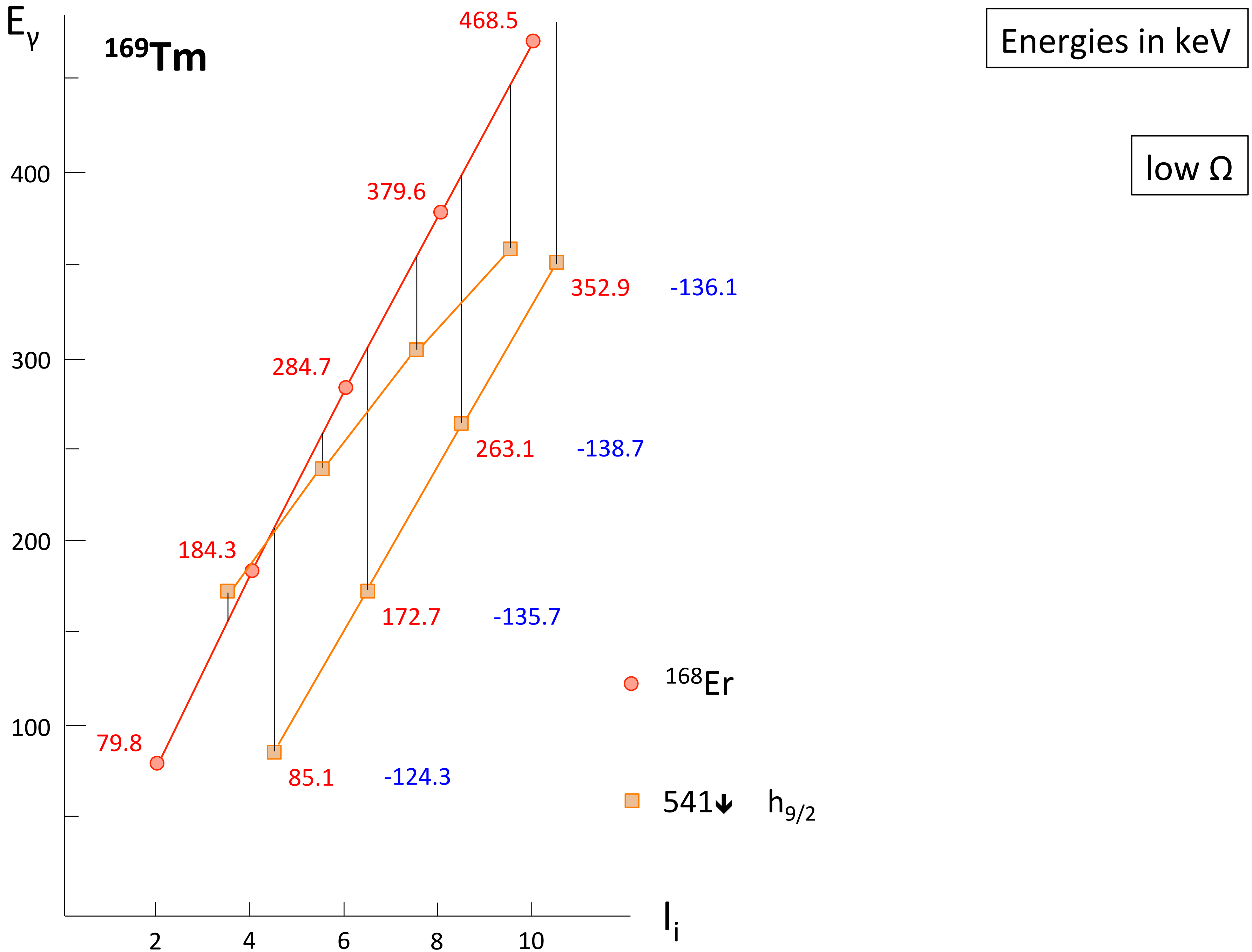


Energies in keV
intermediate Ω
E_γ
^{177}Hf
483.3
82.3
465.8
35.1
401.0
426.5
36.2
94.2
385.0
36.6
341.7
35.8
306.8
296.5
33.9
105.0
249.7
30.9
201.8
84.3
400
300
200
100
2
4
6
8
10
I_i
NOTE: reference
^{176}Hf
514↓
$h_{9/2}$

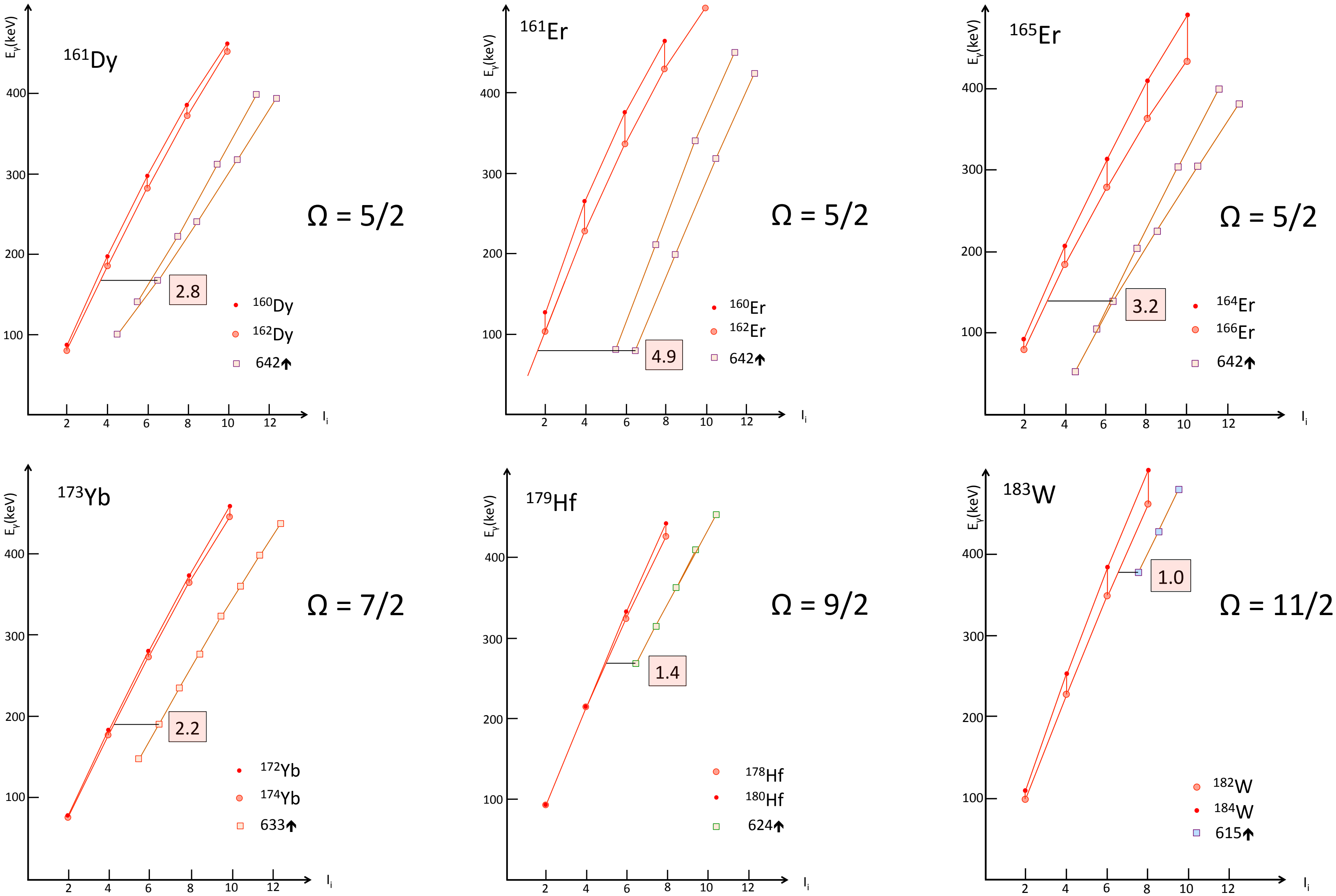


Energies in keV

intermediate Ω



Energies in keV
low Ω
E_γ
^{169}Tm
400
300
200
100
2
4
6
8
10
I_i
468.5
379.6
284.7
184.3
79.8
352.9
-136.1
263.1
-138.7
172.7
-135.7
85.1
-124.3
^{168}Er
541↓ $h_{9/2}$



$\boxed{i_{13/2}}$

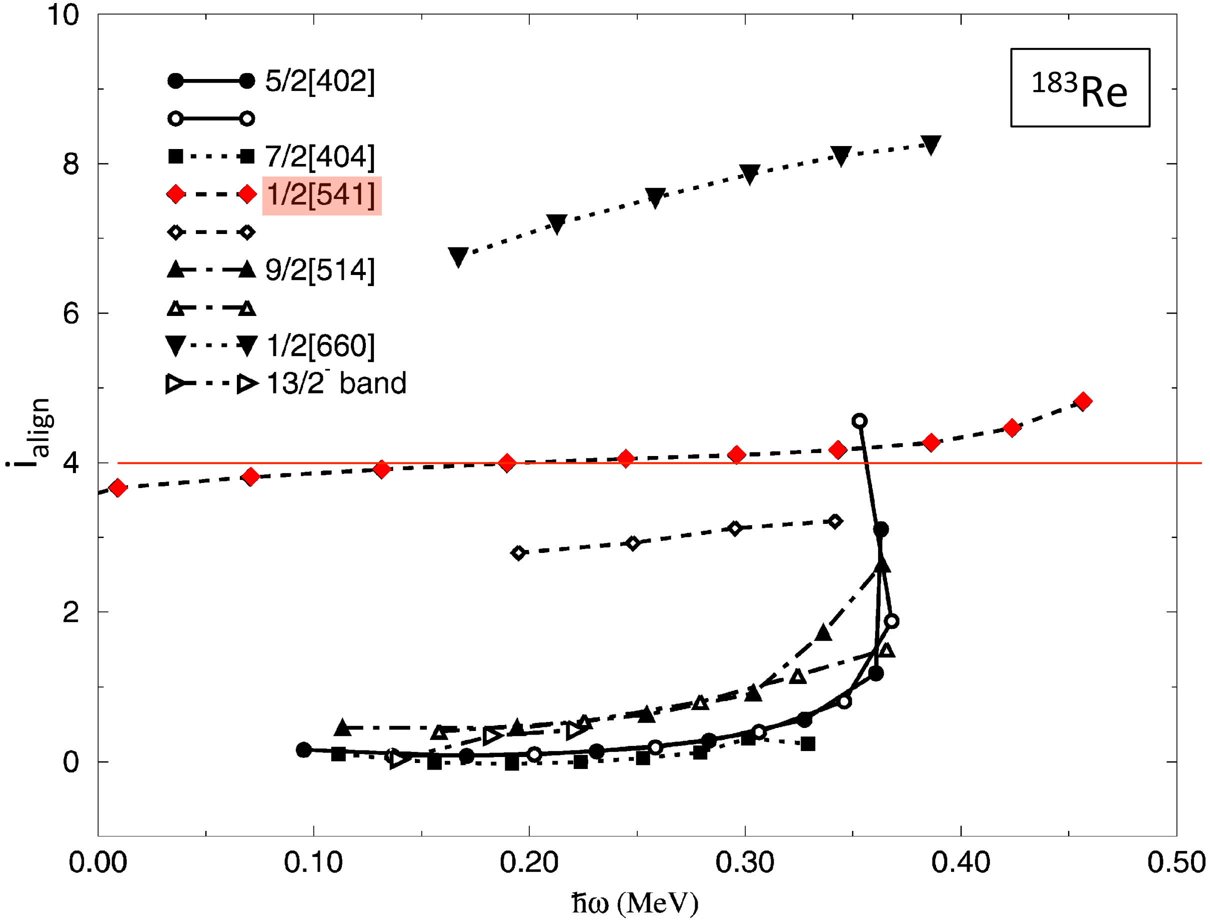
alignment decreases with:
- increasing deformation--increasing M. of I.
- increasing Ω

$$\boxed{-\frac{\hbar^2}{\Im}\hat{\mathbf{I}}\cdot\hat{\mathbf{j}}}$$

“Standard” alignment plots for ^{183}Re

$$i_{Harris} = (\mathcal{J}_0 + \omega^2 \mathcal{J}_1)\omega$$

Harris formula fit to even-even core; then subtract i_{Harris} from I to get i_{align}



Recall:
$p = mv$

$L = I\omega$

Purry
NP A672 54

Alignment of 402 and 404 recognized under name "identical bands"—25 years ago

VOLUME 69, NUMBER 10 PHYSICAL REVIEW LETTERS 7 SEPTEMBER 1992

Low-Spin Identical Bands in Neighboring Odd-A and Even-Even Nuclei: A Possible Challenge to Mean-Field Theories

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A comprehensive study of odd-A rotational bands in normally deformed rare-earth nuclei indicates that a large number of seniority-one configurations (30% for odd-Z nuclei) at low spin have moments of inertia nearly identical to that of the seniority-zero configuration of the neighboring even-even nucleus with one less nucleon. It is difficult to reconcile these results with conventional models of nuclear pair correlation, which predict variations of about 15% in the moments of inertia of configurations differing by one unit in seniority.

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IDENTICAL BANDS IN DEFORMED AND SUPERDEFORMED NUCLEI

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*Seniority = number of unpaired nucleons

Moments of inertia, M. of I. and pairing --sample statements

- "The identical bands are found to be associated with up-sloping particle states, suggesting that the cause may be a cancellation between pairing and deformation decreases."

  --PRL 69 3448 (1992).

- "...BCS [pairing] theory can qualitatively reproduce the experimental large fluctuations in the M. of I. which is helpful to understand the appearance of the normally deformed identical bands in an odd-A nucleus and its even-even neighbors…"

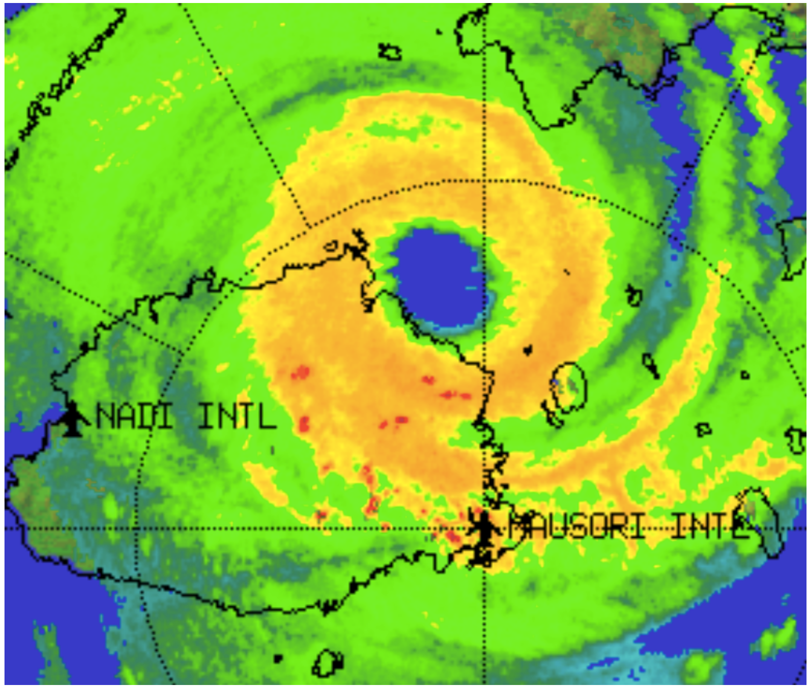
  —PR C63 047306 (2001).

- "In particular, the reduction of the BCS pairing correlations due to the blocking of one and two orbitals implies large changes (up to 30%) in the moments of inertia and cannot be reconciled with these [identical band] systematics."

  --Annu. Rev. Nucl. Part. Sci. 45 485 (1995).

Coriolis Force in Nature

Tropical cyclone Winston:
Southern hemisphere, Fiji



Hurricane Maria:
Northern hemisphere, Dominica